dMAT – Digital Master Assessment Test

Preparatory Materials for Test Takers

dMAT
Digitaler Mastertest

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Digitaler Mastertest

Imprint

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Instructions for the use of the preparation materials for test takers

Dear dMAT participant,

these preparation materials will help you to prepare well for the dMAT exam. Here you get

- general information on the content and structure of the test,
- detailed instructions and hints on how to work through the different task types as well as
- the possibility to work on exercises for each task type in different levels of difficulty including sample solutions.

Read all the information carefully to become well acquainted with the dMAT. The preparation materials are primarily intended to help you to prepare for the exam in terms of content. You can get hints and examples on the digital form of the test in information videos on www.d-mat.de.

Note

The detailed instructions for the individual task types are only available in these preparation materials! In the dMAT exam you will only see short explanations of the processing as a reminder.

We wish you lots of success!

Your dMAT team

General information about the dMAT

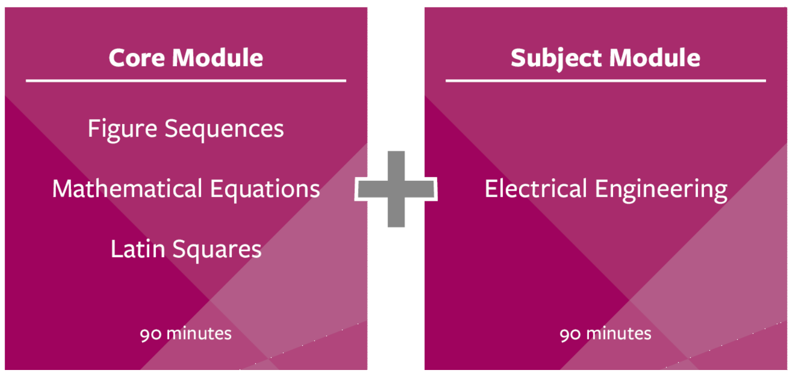
The digital Master Assessment Test (dMAT) is a new study aptitude test used for the admission of international applicants to Master's degree programmes in Germany. Taking the test allows a fair comparison of applicants across different national education systems.

The dMAT is offered digitally and can be taken in both German and English. Developed in close professional collaboration with German universities, the test accurately reflects the requirements of the intended degree programmes and is thus a valid instrument for assessing study ability for master's degree programmes.

The dMAT exams are evaluated centrally at the TestDaF Institute in Bochum. The test is standardized, which ensures that all participants can be compared with each other. In addition, the format of the test is based on the internationally recognized standards for psychodiagnostic tests, the test is reliable and objective.

Structure of the dMAT

The dMAT consists of two parts: A Core Module that tests general study aptitude, and subject-specific modules that test subject-specific aptitude as well as the ability to apply the knowledge acquired in the course of study. Currently, the dMAT is offered for the master's degree programme in electrical engineering, other examination modules are under development. The following graphic illustrates the structure of the test.



The duration of the exam itself is about three hours with a break of 30 minutes between the two parts of the exam.

The **Core Module** measuring general study ability consists of three subtests that measure general cognitive abilities relevant to a master's degree programme in Germany. To a certain extent, the core module allows participants to be compared across the respective subject modules.

The production of the **Subject Modules** is based on extensive scientific studies by experts, so that the exam content is representative of the respective fields of study. The examination tasks are knowledge-based and consist of a combination of a typical subject-related problem (input) and corresponding single-choice questions. The dMAT therefore requires subject knowledge and application skills, but not memorized factual knowledge. You can familiarize yourself with the exact requirements and instructions of the individual task types in the next sections.

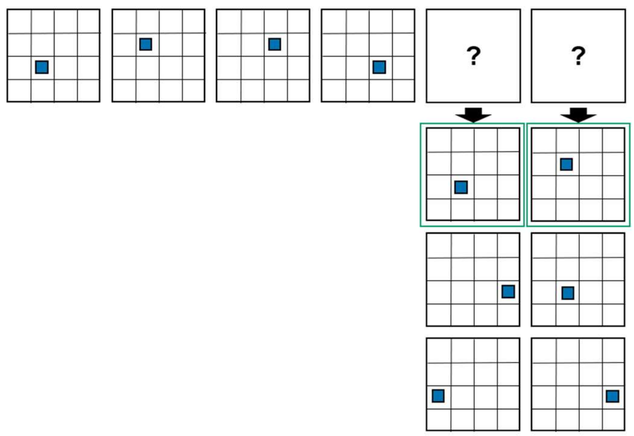
Please note: You may not take notes throughout the exam.

Core Module – Instructions and Exercises

Core Module	Figure Sequences
Instructions	

In this task you will see a series of pictures (matrices). The figures in the matrices can change their **position**, **colour**, and/or **orientation** from one matrix to the next according to specific rules. It is your task to continue the series logically and to determine what the next two matrices look like.

Example Task



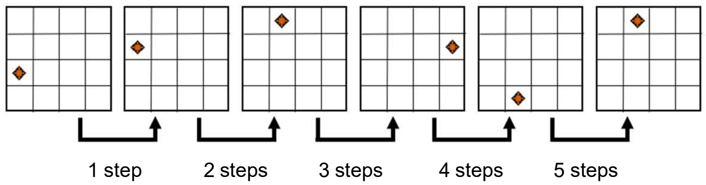
Solution

The blue square always moves one field clockwise within the four middle fields. Therefore, for the fifth matrix the first response option is correct, and for the sixth matrix as well.

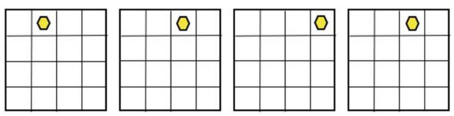
Rules

- Figures can change their colour.
- Figures can rotate around their own axis.
- Figures can move in the matrix. Vertical, horizontal, and diagonal movements are allowed. Figures cannot change from one diagonal movement to another type of movement.

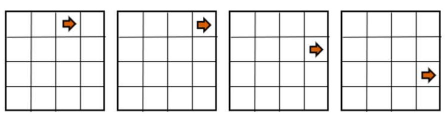
- Figures can also change their movement, colour or orientation by x + 1. Example: If a figure moves one step from matrix 1 to matrix 2, it moves 2 steps from matrix 2 to matrix 3, then 3 steps, etc.



- Figures cannot disappear or overlap.
- Figures cannot leave the matrix. If they come up against an outer boundary, they can EITHER
- bounce off OR



- move along the outer boundary.

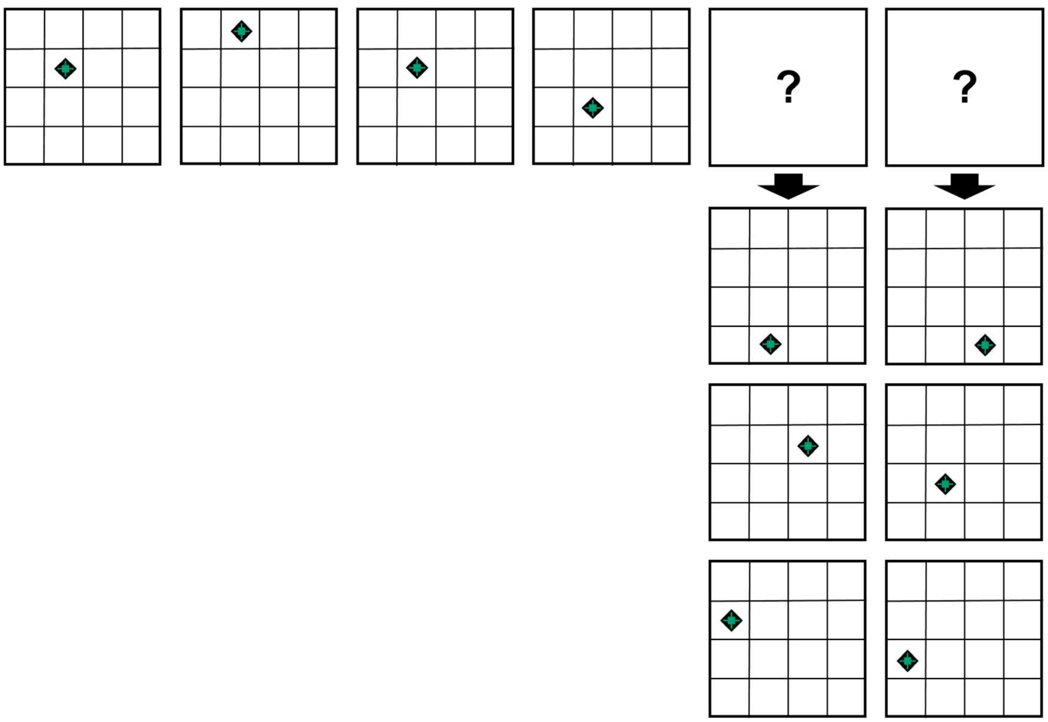


In the exam you have a total of **25 minutes** for **20** series of matrices. Please be as quick and accurate as possible. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module	Figure Sequences
Exercises	

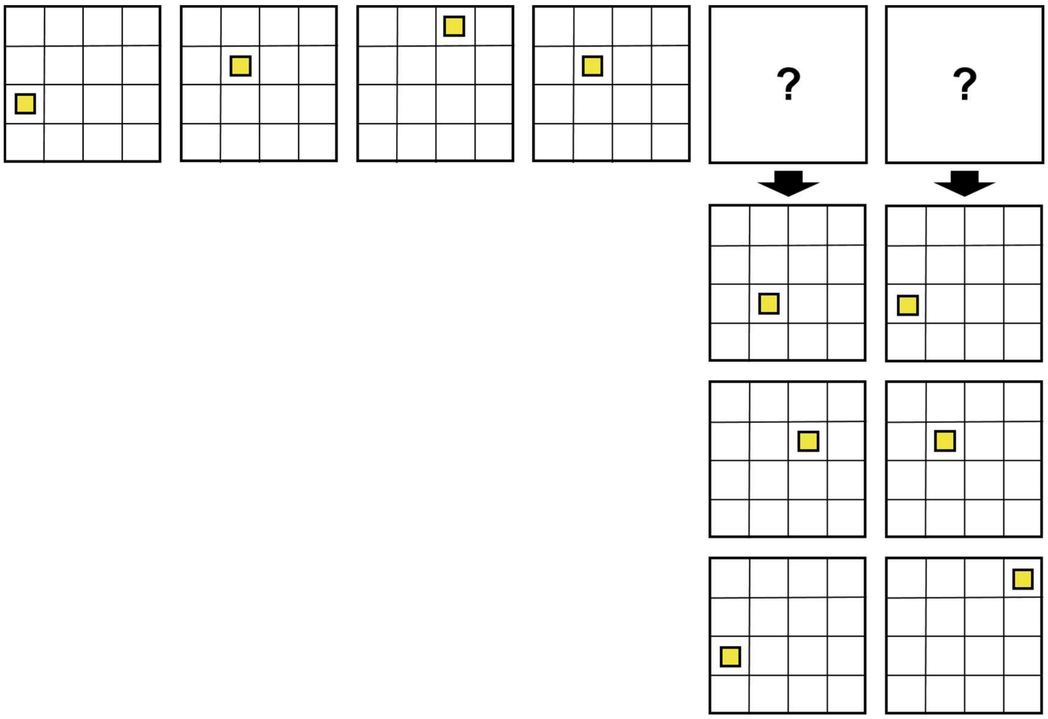
For the task type **Figure Sequences** there are six exercises available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

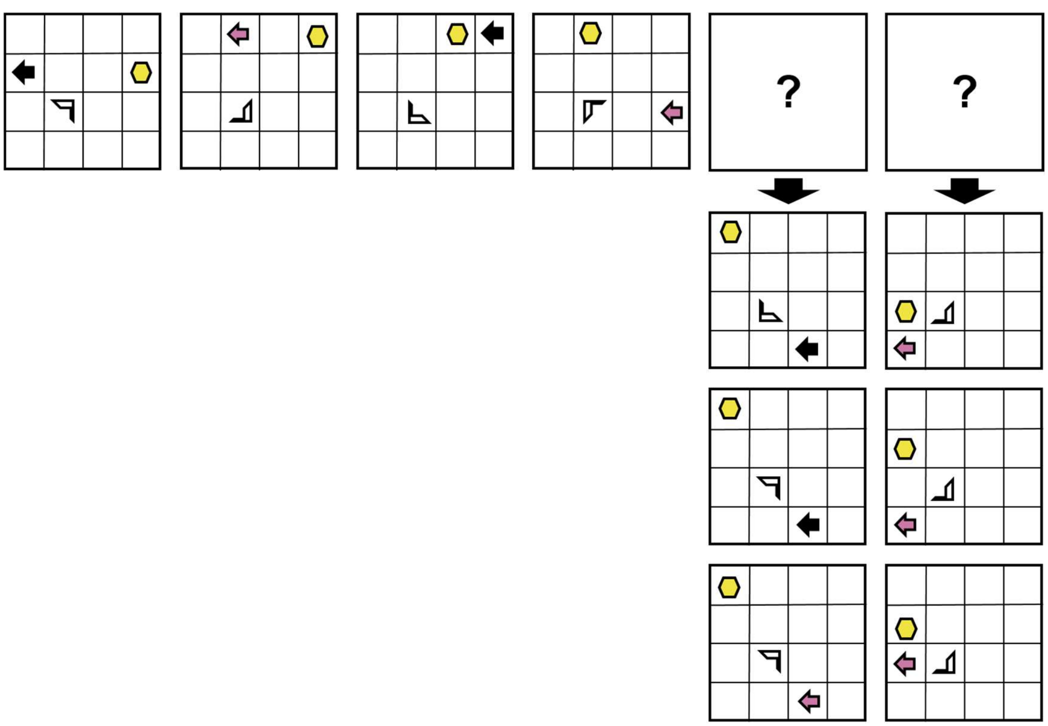


![dMAT Digitaler Mastertest]

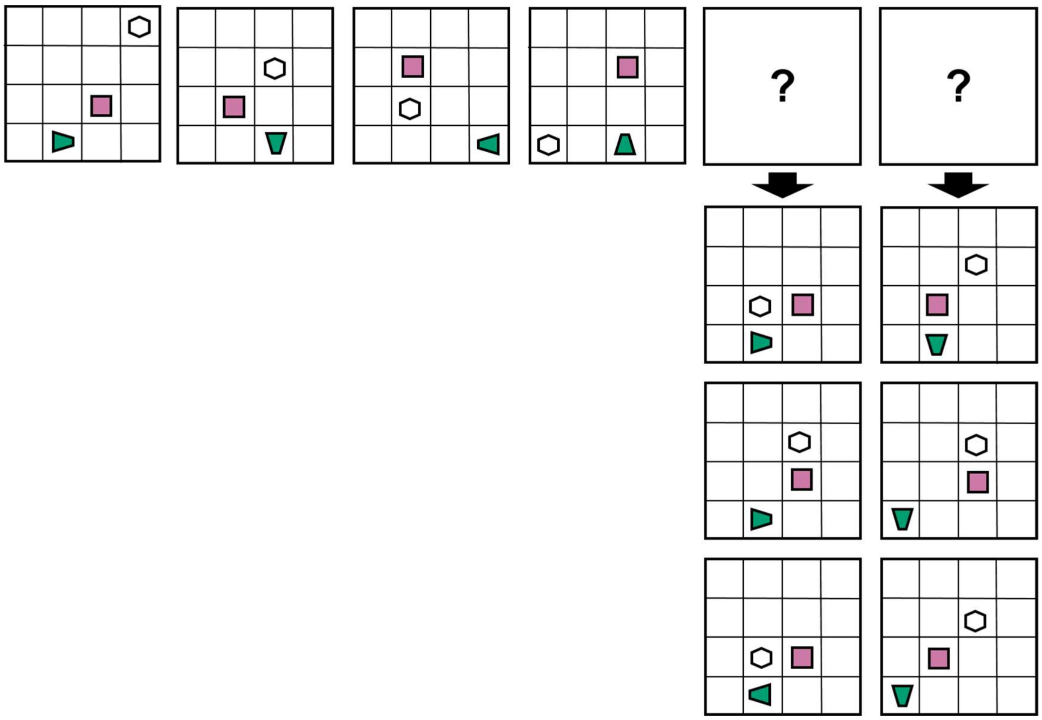
Exercise 2 – Difficulty: low



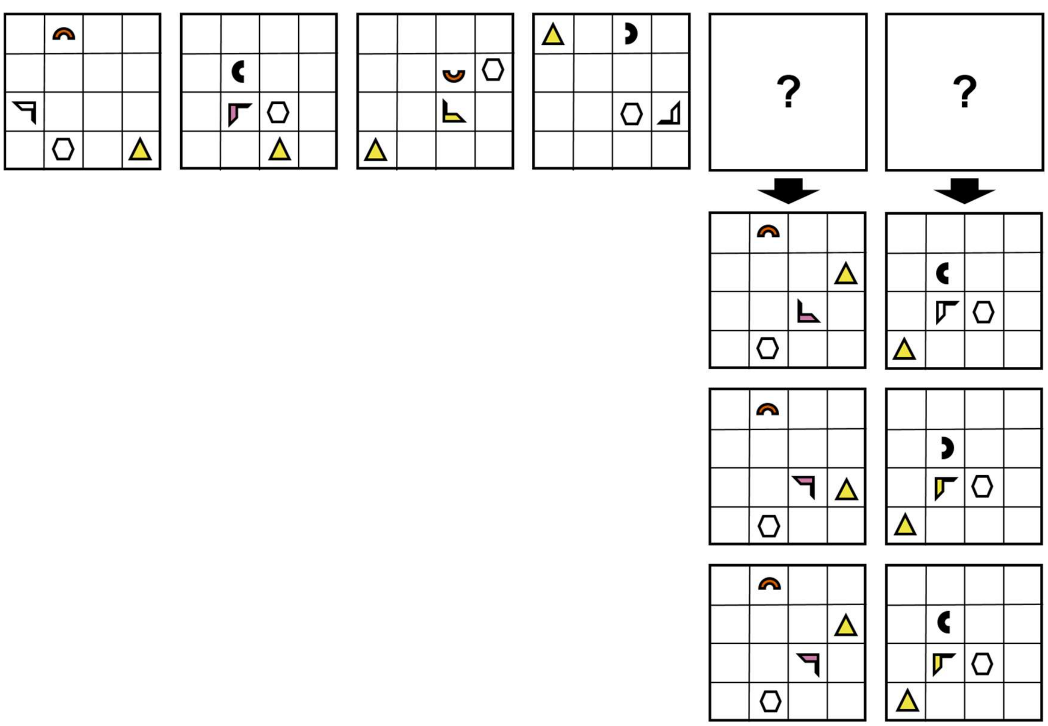
Exercise 3 – Difficulty: medium



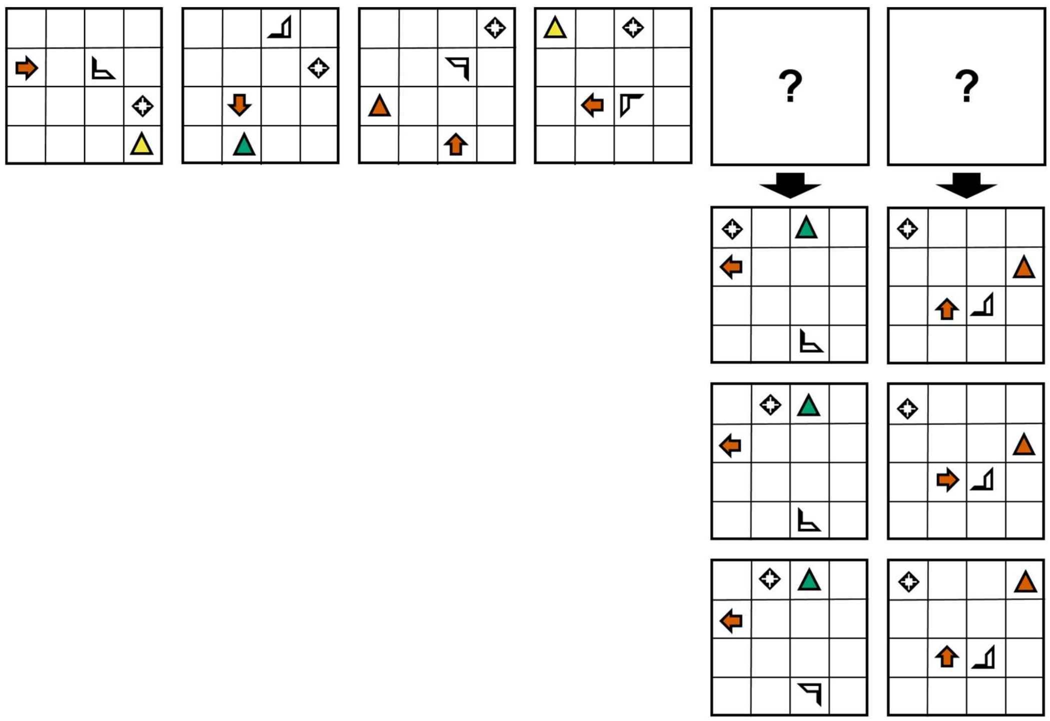
Exercise 4 – Difficulty: medium



Exercise 5 – Difficulty: high

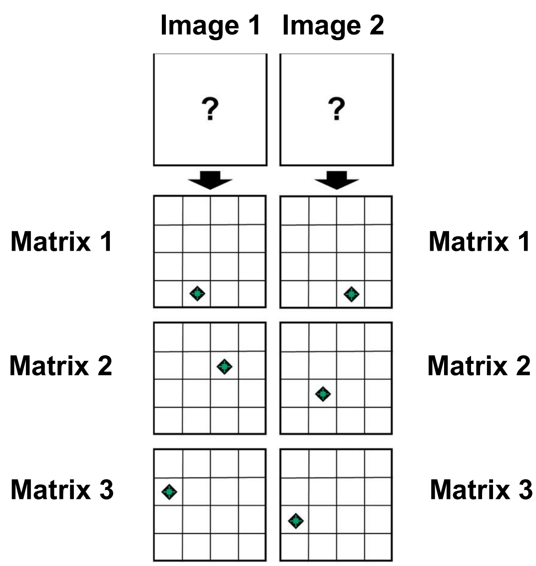


Exercise 6 – Difficulty: high



Core Module	Figure Sequences
Exercises – Solutions	

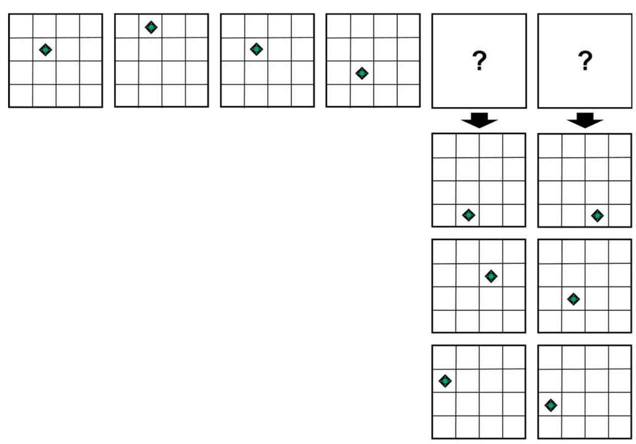
Note on the solution key



Solution – Exercise 1

Image 1: Matrix 1

Image 2: Matrix 2

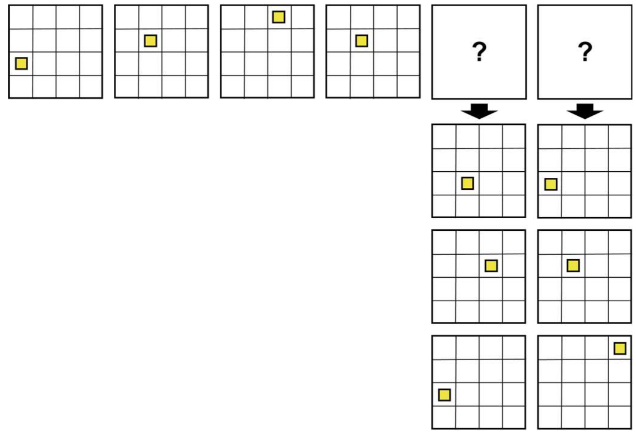


The symbol moves vertically one field at a time in the second column and bounces off the upper or lower boundary.

Solution – Exercise 2

Image 1: Matrix 3

Image 2: Matrix 2

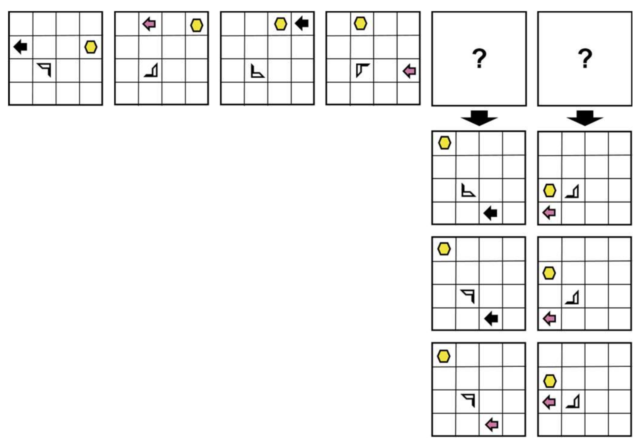


The symbol always moves one space diagonally upwards to the right from its starting position until it bounces off the upper boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 3

Image 1: Matrix 2

Image 2: Matrix 2



The symbol moves along the outer borders clockwise by two squares at a time. It changes its colour alternately from black to pink .

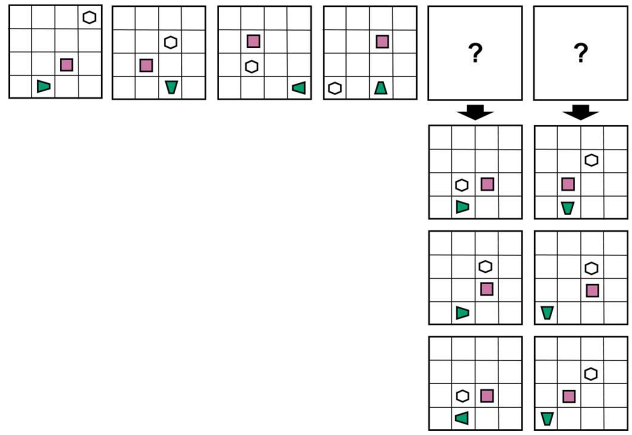
The symbol rotates 90 degrees to the right from image to image.

The symbol moves along the outer borders counter clockwise one space at a time.

Solution – Exercise 4

Image 1: Matrix 1

Image 2: Matrix 3



The symbol moves horizontally by one field in the fourth row and bounces off the right or left border. It rotates 90 degrees to the right from image to image.

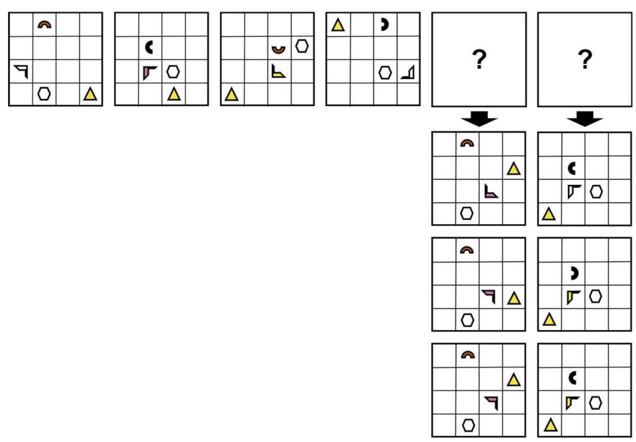
The symbol moves from its starting position one field at a time from image to image. The order of the directions in which the symbol moves is: left, up, right, down, and so on.

The symbol moves from its starting position diagonally downwards to the left until it bounces off the bottom left corner and returns the same way to the top right corner (diagonally upwards to the right).

Solution – Exercise 5

Image 1: Matrix 3

Image 2: Matrix 3



The symbol moves along the outer borders clockwise by x + 1 fields (i.e. from matrix 1 to matrix 2 one field, from matrix 2 to matrix 3 two fields, and so on).

The symbol moves horizontally by one field in the third row and bounces off the right or left border. In doing so, it turns 90 degrees to the left from image to image and changes its colour from white to pink to yellow, and so on.

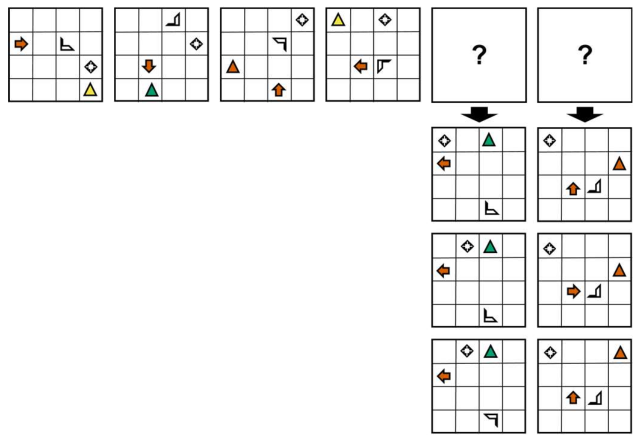
The symbol moves from its starting position one field at a time from image to image. The sequence of directions in which the symbol moves is: down, right, up, left, and so on. It turns 90 degrees to the left and changes its colour alternately from orange to black.

The symbol moves diagonally upwards to the right from its starting position until it bounces off the right boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 6

Image 1: Matrix 2

Image 2: Matrix 1



The symbol moves from its starting position diagonally downwards to the right until it bounces off the lower boundary and returns to the starting position in the same way (diagonally upwards to the left). Once there, it bounces off the left boundary and moves diagonally down to the right again. The symbol always turns x + 1 times to the right by 90 degrees. I.e. from matrix 1 to matrix 2 it rotates once by 90 degrees to the right. From matrix 2 to matrix 3 it rotates twice by 90 degrees to the right, and so on.

The symbol moves vertically by one field in the third column and bounces off the upper or lower border. It rotates 90 degrees to the left from image to image.

The symbol moves along the outer borders counter clockwise one space at a time.

The symbol moves along the outer borders clockwise by two spaces at a time. In the process, it changes colour from yellow to green to orange, etc.

Core Module	Mathematical Equations
Instructions	

In this task, you are supposed to solve systems of equations in such a way that all requirements are met. One system of equations always consists of several single equations.

Your task is to find out which numbers must be used for the unknowns (letters) in the equations so that all equations are correct.

There is always only one solution for each letter, in which all requirements are met.

Each letter can have be an integer between 1 and 20.

Example 1

A + 2 = **B**

B = 6

What number does **A** correspond to so that the equations are solved correctly?

Solution of Example 1

Because of the second equation, you know that **B** = 6. Replace **B** with the number 6 in the first equation and you get **A** + 2 = 6. Solve the first equation and you get **A** = 6 – 2 = 4. Therefore, the solution of the first example is **A** = 4. Any other solution is wrong.

Example 2

B = 2 × **A**

B + **A** = 12

What numbers do **A** and **B** correspond to so that the equations are solved correctly?

Solution of Example 2

The first equation defines that **B** = 2 × **A**. Putting this information in the second equation gives 2 × **A** + **A** = 12 or 3 × **A** = 12. Rearranging this equation gives **A** = 12 ÷ 3 = 4. Putting the number 4 into the first or second equation for **A** gives **B** = 8. Therefore, the solution of the second example is **A** = 4 and **B** = 8. Any other solution is wrong.

In the exam you have **25 minutes** to solve **20 systems of equations**. Please be as quick and accurate as possible. You are not allowed to take notes in the exam.

Core Module	Mathematical Equations
Exercises	

For the task type **Mathematical Equations**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

$$7 + A = 14$$

$$B - 3 = A$$

Exercise 2 – Difficulty: low

$$B \div 2 = A$$

$$B - A = 8$$

Exercise 3 – Difficulty: medium

$$3 \times C = A$$

$$A + C = 8$$

$$2 \times A + 2 \times C = B$$

Exercise 4 – Difficulty: medium

$$18 - B = A$$

$$3 \times A = C$$

$$B \div 2 = A$$

Exercise 5 – Difficulty: high

$$A - B + C - D = 2$$

$$10 \times B = C$$

$$5 \times B = A$$

$$11 + B = D$$

Exercise 6 – Difficulty: high

$$C + D - A = 1$$

$$5 \times C = D$$

$$13 - C = A$$

$$3 \times C - 1 = B$$

Core Module	Mathematical Equations
Exercises – Solutions	

Solution – Exercise 1

$7 + A = 14$
$B – 3 = A$

$A = 7$
$B = 10$

The first equation makes it clear that $A = 7$ if you subtract 7 on both sides. If you insert this information into the second equation, you get $B - 3 = 7$. If you add 3 on both sides, you get the solution $B = 10$.

Solution – Exercise 2

$B \div 2 = A$
$B – A = 8$

$A = 8$
$B = 16$

Multiplying by 2 on both sides in the first equation gives $B = 2A$. Replacing the variable B in the second equation with this information gives $2A - A = 8$. This means $A = 8$. Substituting the solution for A in the first equation gives $B \div 2 = 8$. Multiplying both sides by 2 gives $B = 16$.

Solution – Exercise 3

$3 \times C = A$
$A + C = 8$
$2 \times A + 2 \times C = B$

$A = 6$
$B = 16$
$C = 2$

With the information from the first equation ($3 \times C = A$ or $A = 3C$), A can be replaced in the second equation so that it can be solved for C: $3C + C = 8$ or $4C = 8$. If you divide by 4 on both sides, you get $C = 2$. Thus, the solution of A can be calculated by substituting the value of C into the first equation: $3 \times 2 = A$. Therefore, $A = 6$. By substituting the solutions for A and C, the third equation can be solved for B: $2 \times 6 + 2 \times 2 = B$. Therefore, $B = 16$.

Solution – Exercise 4

$18 - B = A$
$3 \times A = C$
$B \div 2 = A$

$A = 6$
$B = 12$
$C = 18$

If you multiply by 2 on both sides in the third equation, you get $B = 2A$ (alternatively, you can also continue the calculation with $0.5B = A$, for example). If you replace B with this information in the first equation, you get $18 - 2A = A$. If you add 2A on both sides, you get $18 = 3A$. If you now divide by 3, you get $A = 6$. This information can be inserted into the third equation, so that you get $B \div 2 = 6$. If you multiply by 2 on both sides, you get $B = 12$. If you insert the result for A into the second equation, you get $3 \times 6 = C$, so $C = 18$.

Solution – Exercise 5

$A - B + C - D = 2$
$10 \times B = C$
$5 \times B = A$
$11 + B = D$

$A = 5$
$B = 1$
$C = 10$
$D = 12$

The information given in equations two, three and four for the variables A, B and C can be inserted into the first equation so that it can be solved for B: $5B - B + 10B - (11 + B) = 2$. If you dissolve the bracket, you get $5B - B + 10\,B - 11 - B = 2$ or $13B - 11 = 2$. If you add 11 on both sides, you get $13B = 13$. If you divide by 13, you get the solution $B = 1$. This information can be inserted into the other equations and solved for the respective missing variable: $10 \times 1 = C$ or $C = 10$, $5 \times 1 = A$ or $A = 5$ and $11 + 1 = D$ or $D = 12$.

Solution – Exercise 6

$C + D - A = 1$
$5 \times C = D$
$13 - C = A$
$3 \times C - 1 = B$

$A = 11$
$B = 5$
$C = 2$
$D = 10$

The information given in equations two and three for the variables A and D can be inserted into the first equation, so that it can be solved for C: $C + 5C - (13 - C) = 1$. Dissolving the bracket gives $C + 5C - 13 + C = 1$ or $7C - 13 = 1$. Adding 13 on both sides gives $7C = 14$. Dividing by 7 gives the solution $C = 2$. This information can be inserted into the other equations and solved for the respective missing variable: $5 \times 2 = D$ or $D = 10$, $13 - 2 = A$ or $A = 11$ and $3 \times 2 - 1 = B$ or $B = 5$.

Core Module **Latin Squares**

Instructions

In this task you will see a 5x5 grid (a square containing 5 rows and 5 columns).

Some fields of the grid contain letters. Each letter can only appear once in each row and each column. Only the letters that are shown as response options (the row next to the grid) can appear in the grid.

Your task is to decide which letter belongs in the field with the question mark. Sometimes you need to fill in other fields in your mind before you can figure out what letter should replace the question mark.

If you know what the correct solution for the question mark field is, click on the correct response in the solution row.

	?			
	A			
	E			
	D			
C	B			

A
B
C
D
E

Next, you will see two examples.

Example 1

			D	
			A	
			C	
C	D	A	?	
			E	D

A
B
C
D
E

→

			D	
			A	
			C	
C	D	A	B	
			E	D

A
B
C
D
E

Solution of Example 1

In the first example, “B” needs to replace the red question mark, because all other letters D, A, C, and E already appear in this column.

Example 2

A	E	C	D	
				A
				E
				?
	B			C

A
B
C
D
E

→

A	E	C	D	B
				A
				E
				D
	B			C

A
B
C
D
E

Solution of Example 1

In the second example, you first need to fill in “B“ in the first row of the last column. “B” is the only letter, which does not already appear in this row and column. Then you can replace the question mark with “D”, because it is the only letter that does not appear in the last column.

In the exam you have 20 minutes for 16 tasks. Please be as quick and accurate as possible! If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module	Latin Squares
Exercises – Solutions	

For the task type **Latin Squares**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Exercise 2 – Difficulty: low

		?		
			D	A
		E		D
A	D			B
D	B		C	

Exercise 3 – Difficulty: medium

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Exercise 4 – Difficulty: medium

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Exercise 5 – Difficulty: high

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Exercise 6 – Difficulty: high

?				C
	D	E	B	A
B		D	A	
	B	C		D

Note on the solution key

	α	β	γ	δ	ε
1	B	?	A	D	
2	A	B	E	C	
3		A			
4	C				
5	D	E		B	

Solution – Exercise 1

Solution = C

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Solution path:

- In column β, C and D are missing.
- C is already in row 4, so D must be inserted in β4.
- Consequently, C must be inserted in the place of the question mark.

Solution – Exercise 2

Solution = D

		?		
			D	A
		E		D
A	D			B
D	B		C	

Solution path:

- In the place of the question mark, D must be inserted because D is already given in all other columns and rows.

Solution – Exercise 3

Solution = B

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Solution path:

- In column γ, B and C are missing. At position γ4, only B can be inserted, since there is already a B in row 1. This also applies in γ4, or line 4 vice versa for C. Consequently, only a C can be inserted in γ1.
- A and D are missing in column β. A can only be in position β5, because A is already present in row 1. Consequently, only a D can be in position β1.
- From this follows that only one E can be inserted in ε1.
- In row 3 it is now noticeable that A can only be in position ε3, as it is already present in all columns and rows.

- Since a B still has to be inserted in column ε, and there is already a B in line 2, it can only be inserted at the position of the question mark.

Solution – Exercise 4

Solution = D

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Solution path:

- A and D are missing in the first row. A can only be inserted at position α1, since it is already in column γ. Consequently, D must be in position γ1.
- It is now noticeable that D is already present in four different rows and columns and can thus only be used in the place of the question mark.

Solution – Exercise 5

Solution = D

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Solution path:

- Only C can be inserted at position γ4.
- In row 3, A and D are missing. At position γ3, only an A can be inserted because it is already present in column α. Consequently, only a D can be inserted at position α1.
- Only A can be inserted at position β1.
- Only E can be inserted at position α1, as it is already present in all other rows and columns.
- Furthermore, C and B are missing in row 5, whereby only a C is inserted at position α5, since it is already present in column ε. Consequently, there is a B at position ε5.
- In row 1, D and B are still missing. Since B is already in column ε, B must be inserted at position γ1 and D at position ε1.
- At the position of the question mark, D must be inserted, since all other letters are already present in column γ.

Solution – Exercise 6

Solution = E

?				C
	D	E	B	A
B		D	A	
	B	C		D

Solution path:

- At position α3, only a C can be inserted, since all other letters are already in line 3.
- In row 5, A and E are missing. A must be in position α5 because it is already in column δ. Consequently, E is in position δ5.
- In row 4, C and E are missing. Only a C can be inserted at position β4, as it is already present in column ε. Consequently, there is an E at position ε4.
- At position ε2, only a B can be inserted, as all other letters in column ε are already present.
- In column γ, A and B are missing. At position γ1, only a B can be inserted, since there is already a B in the second row. Consequently, A must be inserted in γ2.
- In the first row, A, D and E must be inserted. A must be inserted in β1 because it is already present in all the other columns. Since E is already present in column δ, it must therefore be inserted in the position of the question mark.

Subject Module – Instructions and Exercises

Subject Module	Electrical Engineering
General Instructions	

In this task type you see a text and a number of questions which you have to answer. There are 4 answer options for each question.

For each question, there is only one correct solution.

The text, the questions and the answer options may contain figures, tables and formulas.

For working on the entire subject test in the exam, you have 90 minutes in total. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

For practice and illustration of the subject module tasks, two exercises are available here.

Subject Module	Electrical Engineering
Exercise 1	

Series and Parallel Connections of Ohmic Resistors

Resistors can be connected in series, in parallel, or in any combination thereof.

Series connections

If n resistors with the resistance values $R_1, R_2, R_3, \dots R_n$ are connected in series, the resulting total resistance R_{res} is:

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \dots + R_{\text{n}} = \sum_{i=1}^{n} R_{\text{i}}\,.$$

In special cases the equation can be simplified.

In the case that all n resistors have the same resistance value R, you get:

$$R_{\text{tot}} = n \cdot R\,.$$

In the case of two resistors with the value ratio $R_2 = k \cdot R_1$ you can write:

$$R_{\text{tot}} = R_1 + R_2 = R_1 + k \cdot R_1 = (1+k) \cdot R_1 \quad \text{or} \quad R_{\text{tot}} = R_1 + R_2 = \frac{R_2}{k} + R_2 = \frac{k+1}{k} R_2\,.$$

If the total voltage U_{tot} is applied to the series connection of the n resistors, then the following applies to the voltage U_{x} at a single resistor R_{x} (with $x = 1, 2, 3 \dots n$):

$$U_{\text{x}} = \frac{R_{\text{x}}}{\sum_{i=1}^{n} R_{\text{i}}} U_{\text{tot}}$$

(= formula for so-called “ohmic voltage divider”).

Parallel connections

If the n resistors with the resistance values $R_1, R_2, R_3, \dots R_n$ are connected in parallel, the following applies for the resulting total resistance R_{tot}:

$$R_{\text{tot}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_{\text{n}}}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_{\text{i}}}}$$

This formula simplifies for only two parallel resistors with the value R_1 and R_2 to:

$$R_{\text{tot}} = \frac{R_1 \cdot R_2}{R_1 + R_2}\,.$$

In special cases the equation can be simplified:

If all n resistors have the same resistance value R, you get:

$$R_{\text{tot}} = \frac{1}{n} R .$$

In the case of two resistors with the value ratio $R_2 = k \cdot R_1$, you can write:

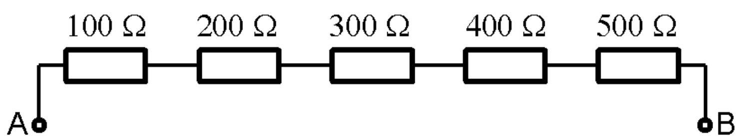
$$R_{\text{tot}} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{R_1 \cdot k \cdot R_1}{R_1 + k \cdot R_1} = \frac{k}{1+k} R_1 \quad \text{or} \quad R_{\text{tot}} = \frac{R_2 / k \cdot R_2}{R_2 / k + R_2} = \frac{R_2 \cdot R_2}{(1+k) R_2} = \frac{R_2}{1+k} .$$

Especially if k is an integer, these formulas may simplify a calculation significantly.

If the total current I_{tot} flows through the n parallel resistors with the total resistance value R_{tot}, then the following applies to the current I_{x} through the individual resistor R_{x} (with $x = 1, 2, 3, \ldots n$):

$$I_{\text{X}} = \frac{R_{\text{tot}}}{R_{\text{X}}} I_{\text{tot}} = \frac{1}{R_{\text{X}} \cdot \sum_{i=1}^{n} \frac{1}{R_{\text{i}}}} I_{\text{tot}}$$ (= formula for “ohmic current-divider”).

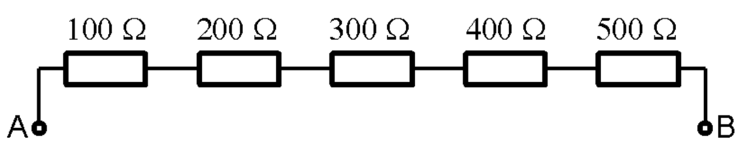
Question 1



What is the value of the total resistance between the terminals A and B?

a) 1000Ω
b) 1200Ω
c) 1400Ω
d) 1500Ω

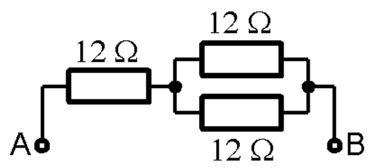
Question 2



There is the voltage $U_{\mathrm{AB}} = 500\,\mathrm{V}$ between terminals A and B.
What voltage occurs at the resistor $R = 300\,\Omega$?

a) $80\,\mathrm{V}$
b) $90\,\mathrm{V}$
c) $100\,\mathrm{V}$
d) $110\,\mathrm{V}$

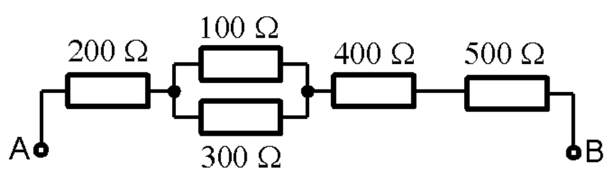
Question 3



What is the value of the total resistance between the terminals A and B?

a) 12Ω
b) 18Ω
c) 24Ω
d) none of the three named resistances

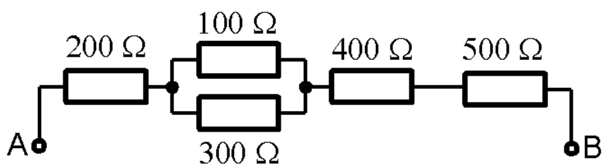
Question 4



What is the value of the total resistance between the terminals A and B?

a) 1125Ω
b) 1150Ω
c) 1175Ω
d) 1200Ω

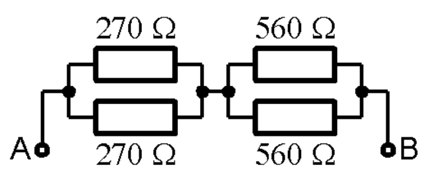
Question 5



There is the voltage $U_{AB} = 1000\,\mathrm{V}$ between terminals A and B.
On which resistor(s) is the voltage $U_R \approx 340\,\mathrm{V}$?

a) $200\,\Omega$
b) $100\,\Omega$ parallel $300\,\Omega$
c) $400\,\Omega$
d) $500\,\Omega$

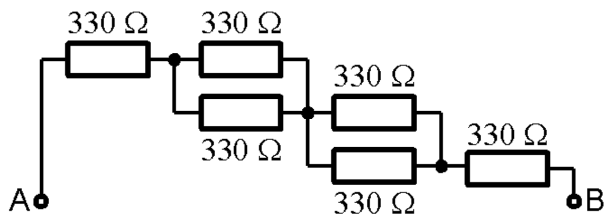
Question 6



What is the value of the total resistance between the terminals A and B?

a) $415\,\Omega$
b) $425\,\Omega$
c) $435\,\Omega$
d) $445\,\Omega$

Question 7



What is the value of the total resistance between the terminals A and B?

a) 960Ω
b) 990Ω
c) 1020Ω
d) 1030Ω

Subject Module	Electrical Engineering
Erxercise 1 – Solutions	

Question 1

Solution: D

In the text it is described that resistors connected in series can be added, that means:

$$500\Omega + 400\Omega + 300\Omega + 200\Omega + 100\Omega = 1500\Omega.$$

Question 2

Solution: C

The formula

$$U_{\mathrm{x}} = \frac{R_{\mathrm{x}}}{\sum_{i=1}^{n} R_{\mathrm{i}}} U_{\mathrm{tot}}$$

shows that the ratio between a resistor R_x and the total resistance corresponds to the ratio between the voltage at resistor R_x and the voltage between terminals A and B. Substituting the resistance $R = 300\Omega$, relative to the total resistance $\sum_{i=1}^{n} R_i = 1500\Omega$, times the total voltage $U_{tot} = 500V$ gives:

$$U_x = \frac{300\Omega}{1500\Omega} \times 500V = \frac{1\Omega}{5\Omega} \times 500V = 100V.$$

Question 3

Solution: B

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies:

$$R_{\text{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 12\Omega$ we get:

$$\frac{1}{2} \times 12\Omega = 6\Omega.$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$12\Omega + 6\Omega = 18\Omega.$$

Question 4

Solution: C

To find the solution, please note that two resistors are connected in parallel here. Since they do not have the same value, the following applies to them:

$$R_{\text{tot}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_\text{n}}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_\text{i}}}$$

By inserting $R_1 = 100\Omega$ and $R_2 = 300\Omega$ we get:

$$R_{tot} = \frac{1}{\frac{1}{100\Omega} + \frac{1}{300\Omega}}$$

Adding the two fractions in the denominator (by bringing them to the same denominator) gives:

$$R_{tot} = \frac{1}{\frac{3}{300\Omega} + \frac{1}{300\Omega}} = \frac{1}{\frac{4}{300\Omega}}$$

By forming the reciprocal you get:

$$R_{tot} = 1 \times \frac{300\Omega}{4} = 75\Omega$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$75\Omega + 200\Omega + 400\Omega + 500\Omega = 1175\Omega.$$

Question 5

Solution: C

The formula

$$U_{\mathrm{x}} = \frac{R_{\mathrm{x}}}{\sum_{i=1}^{n} R_{\mathrm{i}}} U_{\mathrm{tot}}$$

shows that the ratio between a resistor R_x and the total resistance corresponds to the ratio between the voltage at resistor R_x and the voltage between terminals A and B. The required resistance of R_x is therefore $0{,}34 \times$ total resistance (1175Ω). As an approximation, $\frac{1}{3}$ can be used, which quickly makes it clear that only 400Ω can be considered as a solution (the exact value is $399{,}5\Omega$).

Question 6

Solution: A

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies to them:

$$R_{\mathrm{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 270\Omega$, we obtain for the first parallel connection:

$$\frac{1}{2} \times 270\Omega = 135\Omega.$$

For the second parallel connection, the following is obtained by inserting $n = 2$ und $R = 560\Omega$:

$$\frac{1}{2} \times 560\Omega = 280\Omega.$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$135\Omega + 280\Omega = 415\Omega.$$

Question 7

Solution: B

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies to them:

$$R_{\text{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 330\Omega$, the result is for both parallel circuits::

$$\frac{1}{2} \times 330\Omega = 165\Omega.$$

According to the formula for series connection, all resistors can now be added to calculate the total resistance:

$$330\Omega + 165\Omega + 165\Omega + 330\Omega = 990\Omega.$$

Subject Module	Electrical Engineering
Exercise 2	

System Analysis

Control engineering deals with dynamic systems (plants). Dynamic systems can be influenced via a manipulated variable $u(t)$ and have an output signal $y(t)$, which should take on a desired value w. *Figure 1* shows this relationship in the form of the standard control loop. A disturbance $d(t)$ normally acts on the system output.

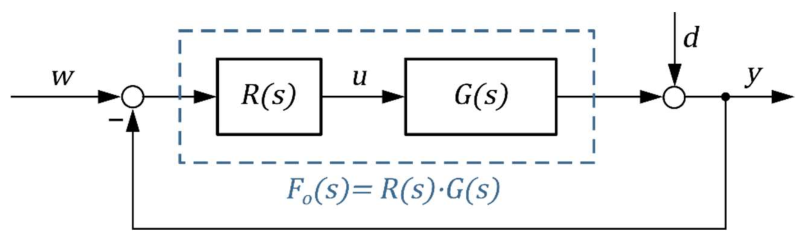


Figure 1: closed loop system – standard control loop

First, the system behavior must be identified and $G(s)$ must be determined so that a controller can be designed that fits the system. To do this, the differential equation describing the system behavior has to be set up first. The differential equation transformed into the Laplace domain provides the system transfer function $G(s)$.

The system transfer function $G(s)$ and a matching function $R(s)$ can be summarized as $F_o(s) = R(s) \cdot G(s)$ (Fig. 1).

$F_o(s)$ denotes the open control loop (index o for open loop) and is a rational function: $F_o(s) = \frac{Z_o(s)}{N_o(s)}$. $Z_o(s)$ denotes the numerator polynomial and $N_o(s)$ the denominator polynomial of $F_o(s)$. The location of the zeros of the numerator polynomial $Z_o(s)$ and the location of the poles (zeros of the denominator polynomial $N_o(s)$) provide information about important properties and the system behavior of the closed loop control system. The most important properties for control engineering are stability as well as steady state accuracy.

The *Nyquist plot* and the *root locus plot* are graphical analysis methods that describe the system behavior via a diagram in the complex number plane. In the Nyquist plot, the transfer behavior of $F_o(s)$ on the imaginary axis is considered for frequencies from $\omega = 0$ to $\omega \to +\infty$, where $s = j\omega$. The graph starts its course at $\omega = 0$.

The root locus plot takes as starting point the positions of the poles and zeros of the open loop $F_o(s)$. The root locus plot shows the position of the poles of the closed loop, if a pure proportional controller (P-controller) $R(s) = K$ is used and K is varied. The root locus plot starts its course in the poles of the open loop.

Question 1

What is the system transfer function $G(s)$ to the differential equation of the form $m \cdot \ddot{y} + c \cdot y - u = 0$?

a) $G(s) = \frac{1}{m \cdot s^2 + c}$
b) $G(s) = \frac{1}{s^2 + m \cdot s + c}$
c) $G(s) = \frac{1}{c \cdot s^2 + m}$
d) $G(s) = \frac{s}{s^2 + m \cdot s + c}$

Question 2

What is the expression for the reference transfer function $F_w(s) = \frac{Y(s)}{W(s)}$, which describes the relationship between the output signal y and the reference signal w in the closed control loop via $F_o(s)$?

a) $F_w(s) = F_o(s)$
b) $F_w(s) = F_o(s) \cdot F_o(s)$
c) $\mathrm{F_w(s)} = \frac{\mathrm{F_o(s)}}{1 + \mathrm{F_o(s)}}$
d) $F_w(s) = \frac{1}{F_o(s)}$

Question 3

What is the expression for the disturbance transfer function $F_d(s) = \frac{Y(s)}{D(s)}$), which describes the relationship between the output signal y and the disturbance signal d in the closed control loop via $F_o(s)$?

a) $F_d(s) = \frac{1}{1+F_o(s)}$
b) $F_d(s) = F_o(s)$
c) $F_d(s) = F_o(s) \cdot F_o(s)$
d) $F_d(s) = \frac{1}{F_o(s)}$

Question 4

Which of the following statements about the reference transfer function and the disturbance transfer function is correct?

a) In a standard control loop, if the reference transfer function has steady state accuracy, the disturbance transfer function has steady state accuracy as well.
b) In a standard control loop, if the reference transfer function is stable, the disturbance transfer function is always stable as well.
c) In the standard control loop, the reference transfer function is equal to the disturbance transfer function.
d) In the standard control loop, the reference transfer function is always stable, while the disturbance function is always unstable.

Question 5

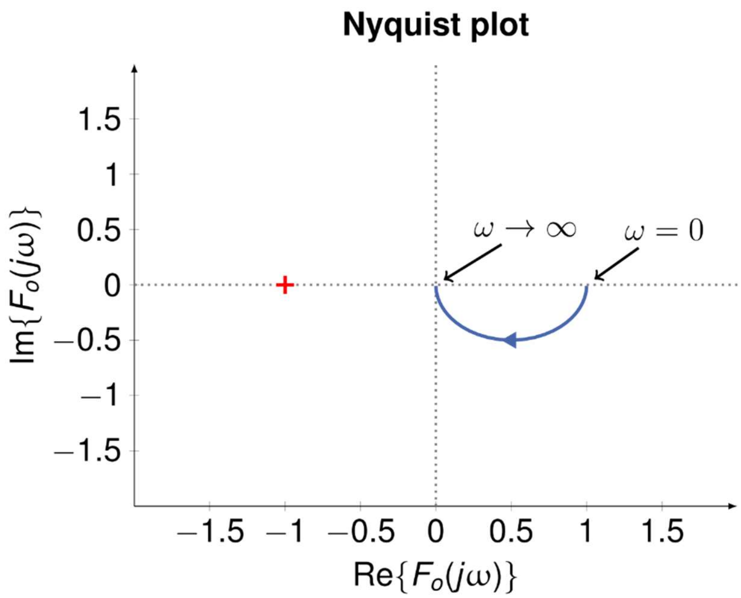


Figure 2: Nyquist plot of an unknown system

What is the transfer function $G(s)$ of the Nyquist plot shown in *Figure 2*?

a) $F_0(s) = \frac{1}{s} = F_0(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega}$

b) $F_0(s) = s = F_0(\mathrm{j}\omega) = \mathrm{j}\omega$

c) $F_0(s) = \frac{1}{s+10} = F_0(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega+10}$

d) $F_0(s) = \frac{1}{s+1} = F_0(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega+1}$

Question 6

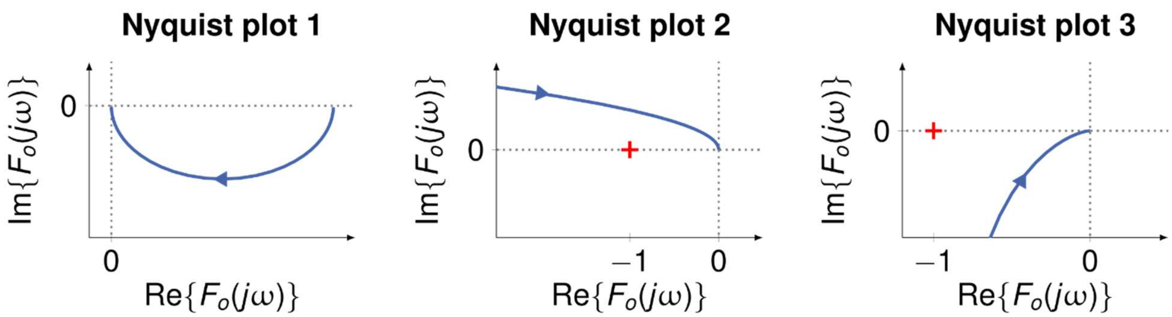


Figure 3: Nyquist plot of three systems 1, 2 and 3

What steady-state behavior is exhibited by the systems whose Nyquist plots are shown in *Figure 3*?

a) System 1: P-behavior, system 2: I²-behavior, system 3: I-behavior
b) System 1: I³-behavior, system 2: I²-behavior, system 3: I-behavior
c) System 1: P-behavior, system 2: P-behavior, system 3: P-behavior
d) System 1: I-behavior, system 2: P-behavior, system 3: I²-behavior

Question 7

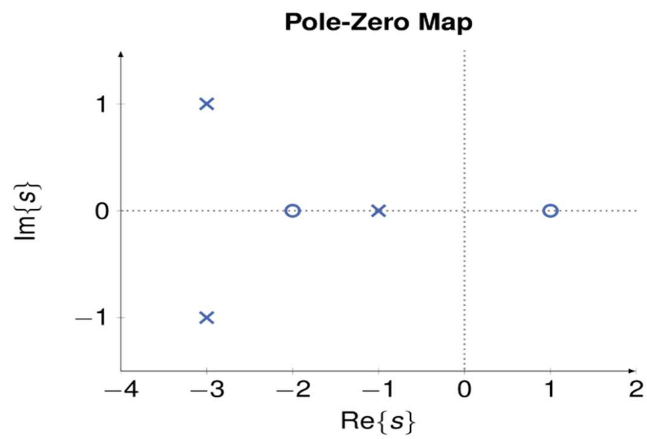


Figure 4: Pole-zero diagram of an unknown system $G(s)$

What is the system function $G(s)$ of the pole-zero diagram shown in *Figure 4*?

a) $G(s) = \frac{(s-1)(s+2)}{(s^2+6s+10)(s+1)}$

b) $G(s) = \frac{(s+1)(s-2)}{(s^2+6s+10)(s+1)}$

c) $G(s) = \frac{(s+1)(s-2)}{(s^2+6s+1\ )(s-1)}$

d) $G(s) = \frac{s-1}{(s^2+6s+10)(s+1)}$

Subject Module	Electrical Engineering
Exercise 2 – Solutions	

Question 1

Solution: A

One describes as usual the Laplace transform

$$\mathcal{L}\{u\}(s) = \int_0^\infty u(t)e^{-st}dt$$

of the function $u(t)$ with $U(s)$ and the Laplace transform $\mathcal{L}\{y\}(s)$ of the function $y(t)$ with $Y(s)$. The Laplace transform of the second derivative $\ddot{y}$ of y is then $\mathcal{L}\{\ddot{y}\}(s) = s^2Y(s)$. The differential equation

$$m \cdot \ddot{y}(t) + c \cdot y(t) - u(t) = 0$$

by applying the Laplace transformation thus goes into the algebraic equation

$$m \cdot s^2 \cdot Y(s) + c \cdot Y(s) - U(s) = 0.$$

By transforming you obtain

$$(m \cdot s^2 + c) \cdot Y(s) = U(s)$$

or

$$Y(s) = \frac{1}{m \cdot s^2 + c} U(s).$$

The relationship between $U(s)$ and $Y(s)$ is given by $Y(s) = G(s)U(s)$, from which

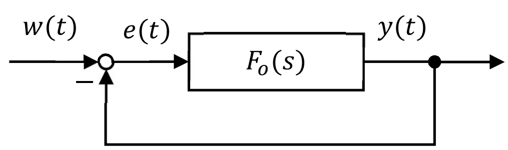
$$G(s) = \frac{1}{m \cdot s^2 + c}$$

follows.

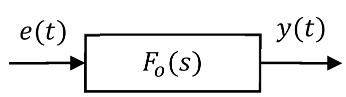
Question 2:

Solution: C

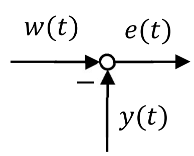
By simplifying the given block diagram and introducing the auxiliary function $e(t)$, you obtain the block diagram



By transition to the Laplace transforms you obtain for the transmission path



the equation $Y(s) = F_o(s)E(s)$ and for the node



the equation $E(s) = W(s) - Y(s)$. Thus the following applies

$$Y(s) = F_o(s)E(s) = F_o(s)\big(W(s) - Y(s)\big).$$

By transforming you obtain

$$Y(s) = F_o(s)W(s) - F_o(s)Y(s)$$

$$\Leftrightarrow Y(s) + F_o(s)Y(s) = F_o(s)W(s)$$

$$\Leftrightarrow (1 + F_o(s))Y(s) = F_o(s)W(s)$$

$$\Leftrightarrow Y(s) = \frac{F_o(s)}{1 + F_o(s)}W(s)$$

$$\Leftrightarrow \frac{Y(s)}{W(s)} = \frac{F_o(s)}{1 + F_o(s)}$$

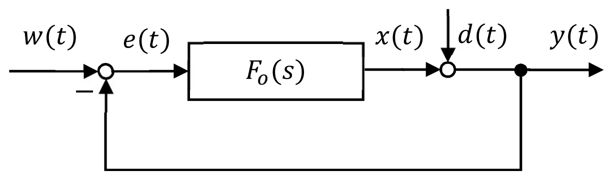
So that means

$$F_w(s) = \frac{F_o(s)}{1 + F_o(s)}$$

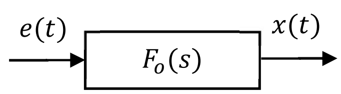
Question 3:

Solution: A

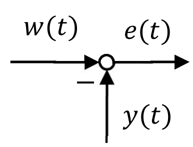
By simplifying the given block diagram and introducing the auxiliary functions $e(t)$ and $x(t)$ you obtain the block diagram



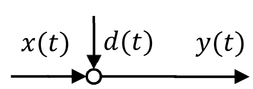
By transition to the Laplace transforms you obtain for the transmission path



the equation $X(s) = F_o(s)E(s)$. For the node



you get the equation $E(s) = W(s) - Y(s)$, and for the node



you get the equation $Y(s) = X(s) + D(s)$. Thus the following applies

$$Y(s) = X(s) + D(s) = F_o(s)E(s) + D(s) = F_o(s)\big(W(s) - Y(s)\big) + D(s).$$

Since this equation must apply independently of the reference variable $w(t)$, we consider the equation in the following for the case $w(t) = 0$. We then obtain

$$Y(s) = -F_o(s)Y(s) + D(s)$$

$$\Leftrightarrow Y(s) + F_o(s)Y(s) = D(s)$$

$$\Leftrightarrow (1 + F_o(s))Y(s) = D(s)$$

$$\Leftrightarrow Y(s) = \frac{1}{1 + F_o(s)} D(s)$$

$$\Leftrightarrow \frac{Y(s)}{D(s)} = \frac{1}{1 + F_o(s)}$$

So that means

$$F_d(s) = \frac{1}{1 + F_o(s)}$$

Question 4

Solution: B

We have shown that the command transfer function $F_w(s)$ is given by

$$F_w(s) = \frac{F_o(s)}{1 + F_o(s)}$$

and the disturbance transfer function is given by

$$F_d(s) = \frac{1}{1 + F_o(s)}$$

With the approach

$$F_o(s) = \frac{Z_o(s)}{N_o(s)}$$

you obtain

$$F_w(s) = \frac{Z_o(s)}{Z_o(s) + N_o(s)}$$

and

$$F_d(s) = \frac{N_o(s)}{Z_o(s) + N_o(s)}$$

where $Z_o(s)$ and $N_o(s)$ are polynomials. The function $Z_o(s) + N_o(s)$ is then also a polynomial. The function $F_w(s)$ is stable exactly when the real parts of all its pole points are negative. The pole places are the zeros of the polynomial $Z_o(s) + N_o(s)$, which are not at the same time zeros of the polynomial $Z_o(s)$. If you now assume that the function $F_d(s)$ is not stable, it follows that there is at least one zero λ of the polynomial $Z_o(s) + N_o(s)$ whose real part is non-negative and which is not at the same time a zero of the polynomial $N_o(s)$. Since according to the prerequisite $Z_o(\lambda) + N_o(\lambda) = 0$, λ cannot be a zero of the polynomial $Z_o(\lambda)$. Thus λ would be a pole point of $F_w(s)$, which contradicts the fact that $F_w(s)$ is stable. Thus, from the stability of $F_w(s)$ always follows the stability of $F_d(s)$.

Question 5

Solution: D

We are looking for the transfer function that yields the value 1 for $\omega = 0$, whose imaginary part is negative for $\omega > 0$ and which tends towards 0 for $\omega \to \infty$. By conjugate complex expansion you obtain for the following representation for the transfer function in d):

$$F_o(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega + 1} = \frac{1 - \mathrm{j}\omega}{(1 + \mathrm{j}\omega)(1 - \mathrm{j}\omega)} = \frac{1 - \mathrm{j}\omega}{1 + \omega^2} = \frac{1}{1 + \omega^2} + \mathrm{j}\frac{-\omega}{1 + \omega^2}$$

So that means

$$\mathrm{Re}\{F_o(\mathrm{j}\omega)\} = \frac{1}{1 + \omega^2},$$

$$\mathrm{Im}\{F_o(\mathrm{j}\omega)\} = \frac{-\omega}{1 + \omega^2}.$$

For $\omega = 0$ you obtain $\mathrm{Re}\{F_o(0)\} = 1$ and $\mathrm{Im}\{F_o(0)\} = 0$. For $\omega > 0$ the real part of $F_o(\mathrm{j}\omega)$ takes positive values between 0 and 1, and the imaginary part of $F_o(\mathrm{j}\omega)$ takes negative values. Furthermore

$$\lim_{\omega \to \infty} \mathrm{Re}\{F_o(\mathrm{j}\omega)\} = 0$$

and

$$\lim_{\omega \to \infty} \mathrm{Im}\{F_o(\mathrm{j}\omega)\} = 0$$

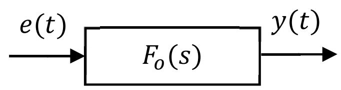
Therefore, the Nyquist plot shown in the graph fits the transfer function

$$F_o(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega + 1}.$$

Question 6

Solution: B

We consider a transmission system of the form



with input signal $e(t)$ and output signal $y(t)$. Then $Y(s) = F_o(s)E(s)$ applies.

The transmission system shows a P-behaviour if a differential equation of the form

$$y(t) + T\dot{y}(t) = Ke(t)$$

with positive, real constants K and T. By applying the Laplace transformation you obtain

$$Y(s)(1 + sT) = KE(s)$$

and thus

$$F_o(s) = \frac{K}{1 + Ts}.$$

For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{1 + \mathrm{j}T\omega} = \frac{K(1 - \mathrm{j}T\omega)}{(1 + \mathrm{j}T\omega)(1 - \mathrm{j}T\omega)} = \frac{K}{1 + T^2\omega^2} + \mathrm{j}\frac{KT\omega}{1 + T^2\omega^2}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the fourth quadrant of the complex plane, whereby it begins for $\omega = 0$ at the point K for $\omega \to \infty$ towards 0. The Nyquist plot for system 1 takes such a course.

The transmission system shows I-behaviour when an integral differential equation of the form

$$y(t) + T\dot{y}(t) = K\int_0^t e(\tau)\,\mathrm{d}\tau$$

with positive real constants K and T holds. By applying the Laplace transformation, you obtain

$$Y(s)(1 + sT) = \frac{K}{s}E(s)$$

and thus

$$F_o(s) = \frac{K}{\mathrm{s} + Ts^2}.$$

For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{\mathrm{j}\omega - T\omega^2} = \frac{K(-\mathrm{j}\omega - T\omega^2)}{(\mathrm{j}\omega - T\omega^2)(-\mathrm{j}\omega - T\omega^2)} = \frac{-KT\omega^2}{\omega^2 + T^2\omega^4} + \mathrm{j}\frac{-K\omega}{\omega^2 + T^2\omega^4}$$
$$= \frac{-KT}{1 + T^2\omega^2} + \mathrm{j}\frac{-K}{\omega + T^2\omega^3}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the third quadrant of the complex plane, whereby it tends towards 0 for $\omega \to \infty$. The Nyquist plot takes such a course for system 3.

An I²-behaviour exists if the transfer function of the system is of the form

$$F_o(s) = \frac{K}{s^2 + Ts^3}$$

with positive, real constants K and T. For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{-\omega^2 - \mathrm{j}T\omega^3} = \frac{K(-\omega^2 + \mathrm{j}T\omega^3)}{(-\omega^2 - \mathrm{j}T\omega^3)(-\omega^2 + \mathrm{j}T\omega^3)} = \frac{-KT\omega^2}{\omega^4 + T^2\omega^6} + \mathrm{j}\frac{KT\omega^3}{\omega^4 + T^2\omega^6}$$
$$= \frac{-KT}{\omega^2 + T^2\omega^4} + \mathrm{j}\frac{KT}{\omega + T^2\omega^3}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the second quadrant of the complex plane, whereby it tends towards 0 for $\omega \to \infty$. This is the course of the Nyquist plot for system 2.

Question 7

Solution: A

We are looking for the fractional rational transfer function $G(s)$, that has the zeros $z_1 = 1$ und $z_2 = -2$ and the poles $p_1 = -3 + \mathrm{j}$, $p_2 = -3 - \mathrm{j}$ and $p_3 = -1$. According to the fundamental theorem of algebra, such a function is of the form

$$G(s) = \frac{(s - z_1)^{\mu_1}(s - z_2)^{\mu_2}}{(s - p_1)^{\nu_1}(s - p_2)^{\nu_2}(s - p_3)^{\nu_3}} = \frac{(s - 1)^{\mu_1}(s + 2)^{\mu_2}}{(s + (3 - \mathrm{j}))^{\nu_1}(s + (3 + \mathrm{j}))^{\nu_2}(s + 1)^{\nu_3}}.$$

Among the given transfer functions, therefore, only the one mentioned in a) comes into question and that with $\mu_1 = \mu_2 = \nu_1 = \nu_2 = \nu_3 = 1$. In fact, the following applies

$$\begin{aligned}(s + (3 - \mathrm{j}))^1(s + (3 + \mathrm{j}))^1 &= (s + (3 - \mathrm{j}))(s + (3 + \mathrm{j})) \\ &= s^2 + (3 - \mathrm{j})s + (3 + \mathrm{j})s + (3 - \mathrm{j})(3 + \mathrm{j}) = s^2 + 3s - \mathrm{js} + 3s + \mathrm{js} + 3^2 - \mathrm{j}^2 \\ &= s^2 + 6s + 10\end{aligned}$$

and thus

$$G(s) = \frac{(s - 1)^1(s + 2)^1}{(s + (3 - \mathrm{j}))^1(s + (3 + \mathrm{j}))^1(s + 1)^1} = \frac{(s - 1)(s + 2)}{(s^2 + 6s + 10)(s + 1)}.$$

The dMAT is offered by the Gesellschaft für Akademische Studienvorbereitung und Testentwicklung e.V. (Society for Academic Study Preparation and Test Development; g.a.s.t.). The worldwide organisation of the dMAT lies with the TestDaF Institut in Bochum.

The format of the dMAT was developed in cooperation with the Universities of Ulm and Kassel. Partners in the creation of the subject modules are universities in Germany.

The development of the dMAT is supported by the Deutscher Akademischer Austauschdienst (German Academic Exchange Service; DAAD), Bonn.