dMAT – Digital Master Assessment Test
Subject Module Computer Science
for the MSc Computer Science

Preparatory Materials for Test Takers

dMAT
Digitaler Mastertest

Imprint

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Table of contents

Instructions for the use of the preparation materials for test takers

Dear dMAT participant,

these preparation materials will help you to prepare well for the dMAT exam. Here you get

- general information on the content and structure of the test,
- detailed instructions and hints on how to work through the different task types as well as
- the possibility to work on exercises for each task type in different levels of difficulty including sample solutions.

Read all the information carefully to become well acquainted with the dMAT. The preparation materials are primarily intended to help you to prepare for the exam in terms of content. You can get hints and examples on the digital form of the test in information videos on www.d-mat.de.

> **Note**
>
> **The detailed instructions for the individual task types are only available in these preparation materials! In the dMAT exam you will only see short explanations of the processing as a reminder.**

We wish you lots of success!

Your dMAT team

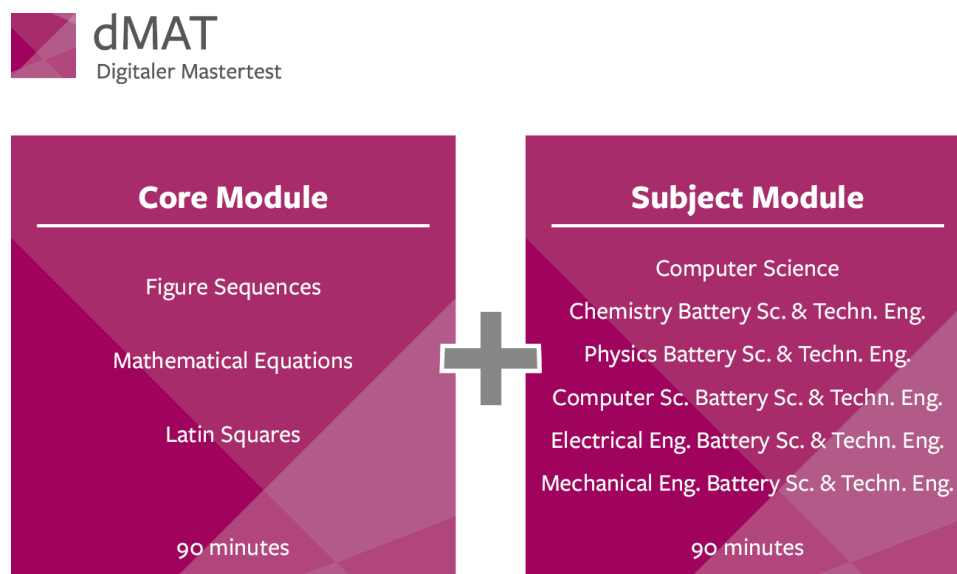# General information about the dMAT

The digital Master Assessment Test (dMAT) is a new study aptitude test used for the admission of international applicants to Master's degree programmes in Germany. Taking the test allows a fair comparison of applicants across different national education systems.

The dMAT is offered digitally. Developed in close professional collaboration with German universities, the test accurately reflects the requirements of the intended degree programmes and is thus a valid instrument for assessing study ability for master's degree programmes.

The dMAT exams are evaluated centrally at the TestDaF Institute in Bochum. The test is standardized, which ensures that all participants can be compared with each other. In addition, the format of the test is based on the internationally recognized standards for psychodiagnostic tests, the test is reliable and objective.

Structure of the dMAT

The dMAT consists of two parts: A Core Module that tests general study aptitude, and subject-specific modules that test subject-specific aptitude as well as the ability to apply the knowledge acquired in the course of study. The following graphic illustrates the structure of the test and the current subject modules.

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Core Module	Subject Module
Figure Sequences	Computer Science
Mathematical Equations	Chemistry Battery Sc. & Techn. Eng.
	Physics Battery Sc. & Techn. Eng.
Latin Squares	Computer Sc. Battery Sc. & Techn. Eng.
	Electrical Eng. Battery Sc. & Techn. Eng.
	Mechanical Eng. Battery Sc. & Techn. Eng.
90 minutes	90 minutes

The duration of the exam itself is about three hours with a break of 30 minutes between the two parts of the exam.

The **Core Module** measuring general study ability consists of three subtests that measure general cognitive abilities relevant to a master's degree programme in Germany. To a certain extent, the core module allows participants to be compared across the respective subject modules.

The production of the **Subject Modules** is based on extensive scientific studies by experts, so that the exam content is representative of the respective fields of study. The examination tasks are knowledge-based and consist of a combination of a typical subject-related problem (input) and corresponding single-choice questions. The dMAT therefore requires subject knowledge and application skills, but not memorized factual knowledge. You can familiarize yourself with the exact requirements and instructions of the individual task types in the next sections.

Please note: You may not take notes throughout the exam.

Core Module – Instructions and Exercises

Core Module	Figure Sequences
Instructions	

In this task you will see a series of pictures (matrices). The figures in the matrices can change their **position**, **colour**, and/or **orientation** from one matrix to the next according to specific rules. It is your task to continue the series logically and to determine what the next two matrices look like.

Example Task
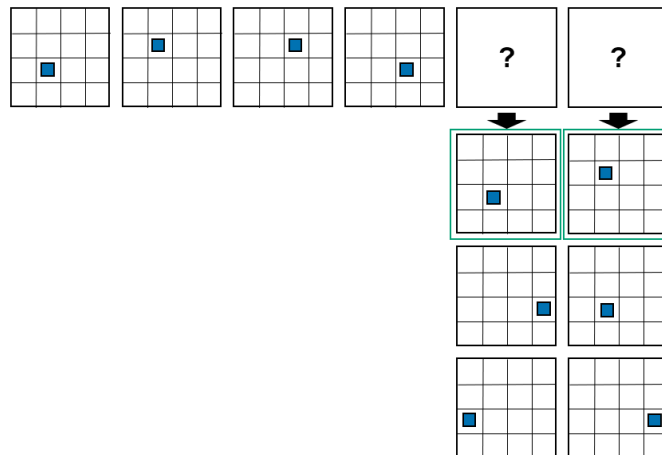
Solution

The blue square always moves one field clockwise within the four middle fields. Therefore, for the fifth matrix the first response option is correct, and for the sixth matrix as well.

Rules

- Figures can change their colour.

- Figures can rotate around their own axis.

- Figures can move in the matrix. Vertical, horizontal, and diagonal movements are allowed. Figures cannot change from one diagonal movement to another type of movement.

- Figures can also change their movement, colour or orientation by x + 1. Example: If a figure moves one step from matrix 1 to matrix 2, it moves 2 steps from matrix 2 to matrix 3, then 3 steps, etc.

1 step   2 steps   3 steps   4 steps   5 steps
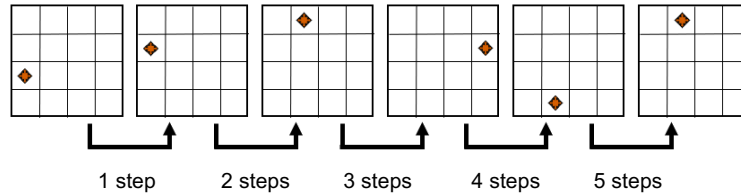
- Figures cannot disappear or overlap.

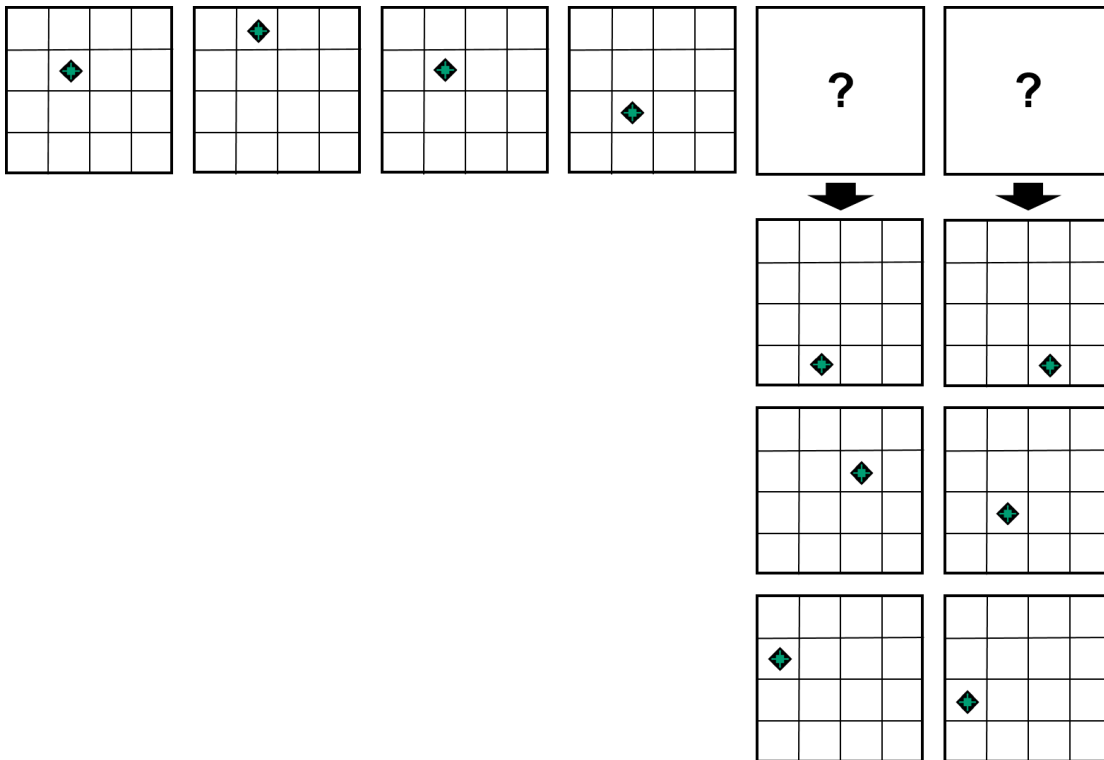- Figures cannot leave the matrix. If they come up against an outer boundary, they can EITHER

- bounce off OR
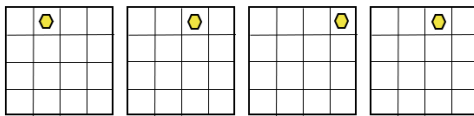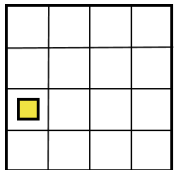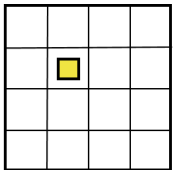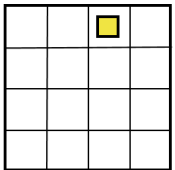
- move along the outer boundary.
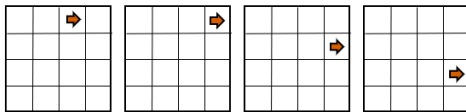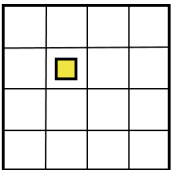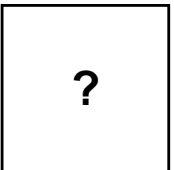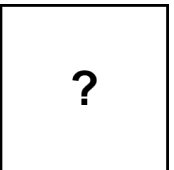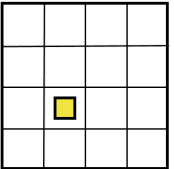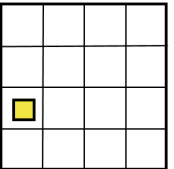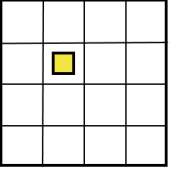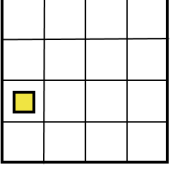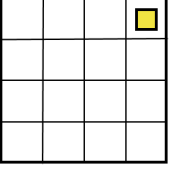
In the exam you have a total of **25 minutes** for **20** series of matrices. Please be as quick and accurate as possible. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module	Figure Sequences
Exercises	

For the task type **Figure Sequences** there are six exercises available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including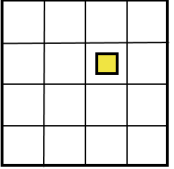 the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.
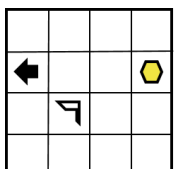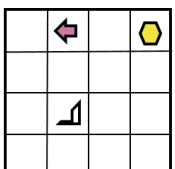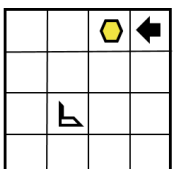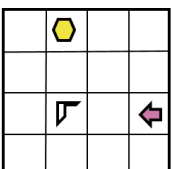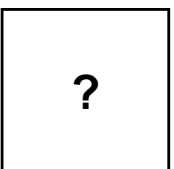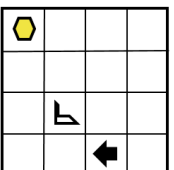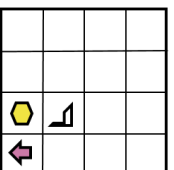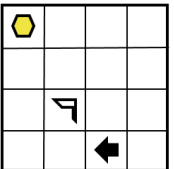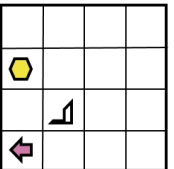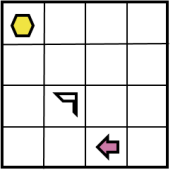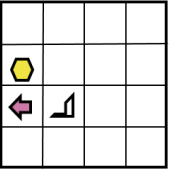
Exercise 1 – Difficulty: low

Exercise 2 – Difficulty: low

Exercise 3 – Difficulty: medium

Exercise 4 – Difficulty: medium

Exercise 5 – Difficulty: high

Exercise 6 – Difficulty: high

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Core Module	Figure Sequences
Exercises – Solutions	

Note on the solution key

Image 1   Image 2

Solution – Exercise 1

Image 1: Matrix 1

Image 2: Matrix 2
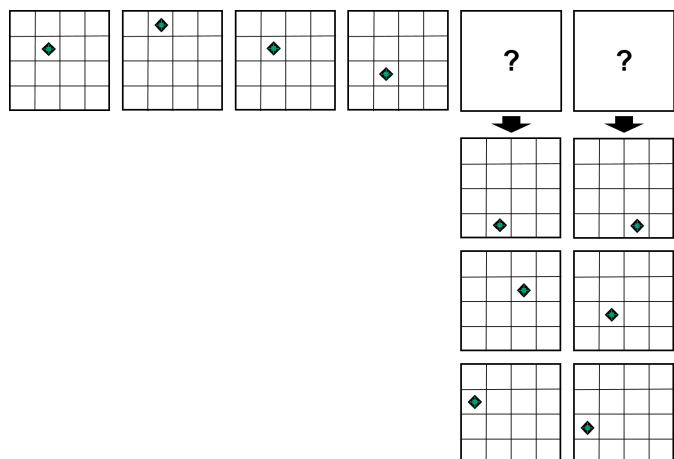
The symbol ◆ moves vertically one field at a time in the second column and bounces off the upper or lower boundary.

Solution – Exercise 2

Image 1: Matrix 3

Image 2: Matrix 2
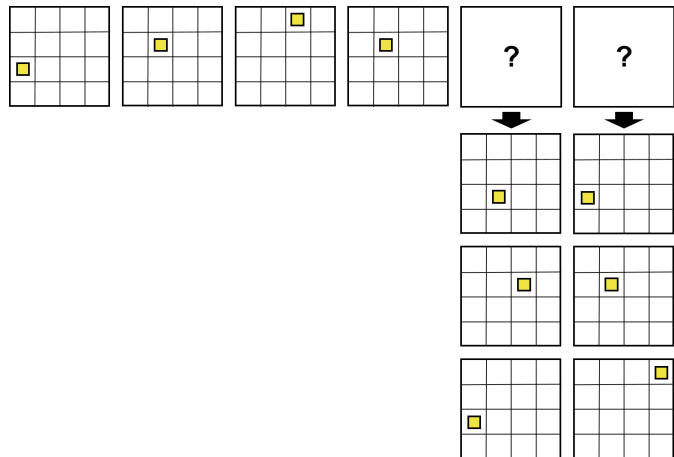
The symbol ⬜ always moves one space diagonally upwards to the right from its starting position until it bounces off the upper boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 3

Image 1: Matrix 2

Image 2: Matrix 2
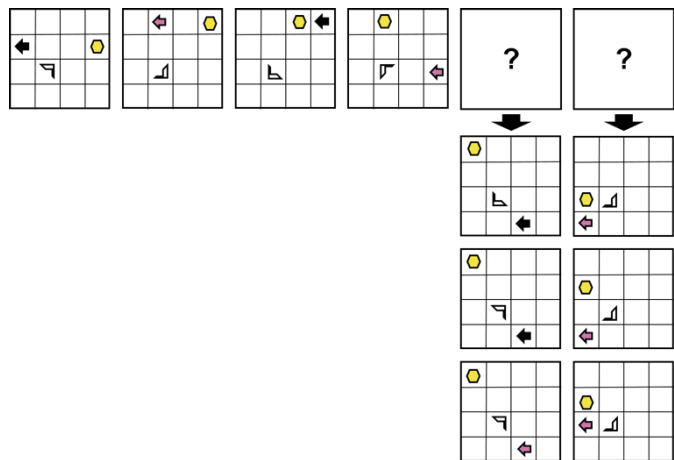
The symbol ⬅ moves along the outer borders clockwise by two squares at a time. It changes its colour alternately from black ⬅ to pink ⬅.

The symbol ⌐ rotates 90 degrees to the right from image to image.

The symbol ⬡ moves along the outer borders counter clockwise one space at a time.

Solution – Exercise 4

Image 1: Matrix 1

Image 2: Matrix 3
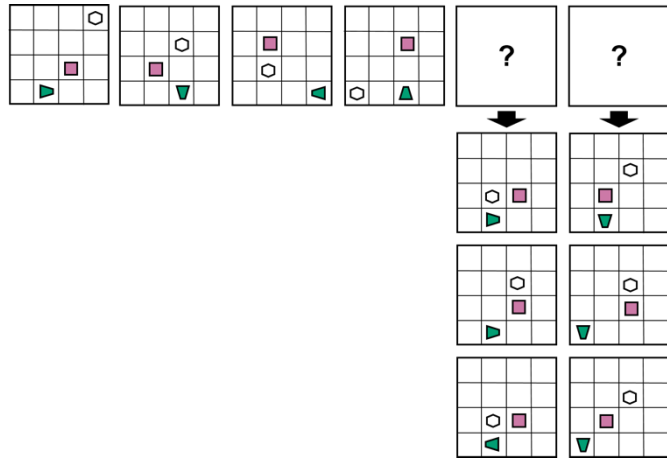
The symbol ▶ moves horizontally by one field in the fourth row and bounces off the right or left border. It rotates 90 degrees to the right from image to image.

The symbol ■ moves from its starting position one field at a time from image to image. The order of the directions in which the symbol moves is: left, up, right, down, and so on.

The symbol ⬡ moves from its starting position diagonally downwards to the left until it bounces off the bottom left corner and returns the same way to the top right corner (diagonally upwards to the right).

Solution – Exercise 5

Image 1: Matrix 3

Image 2: Matrix 3
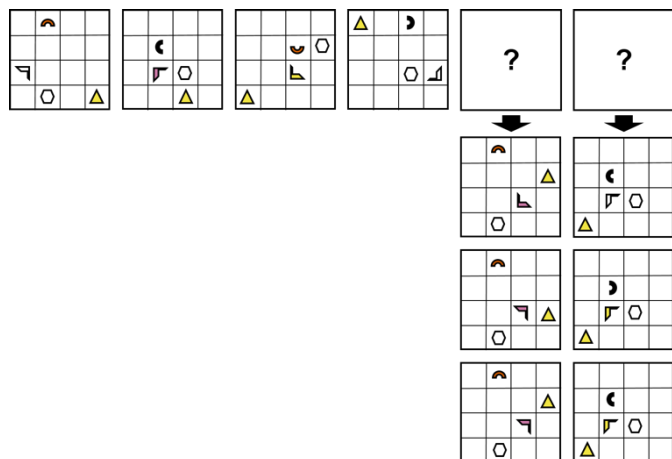
The symbol ▲ moves along the outer borders clockwise by x + 1 fields (i.e. from matrix 1 to matrix 2 one field, from matrix 2 to matrix 3 two fields, and so on).

The symbol � moves horizontally by one field in the third row and bounces off the right or left border. In doing so, it turns 90 degrees to the left from image to image and changes its colour from white ⌐ to pink ⌐ to yellow ⌐, and so on.

The symbol ⌒ moves from its starting position one field at a time from image to image. The sequence of directions in which the symbol moves is: down, right, up, left, and so on. It turns 90 degrees to the left and changes its colour alternately from orange ⌒ to black ⌒.

The symbol ⬡ moves diagonally upwards to the right from its starting position until it bounces off the right boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 6

Image 1: Matrix 2

Image 2: Matrix 1
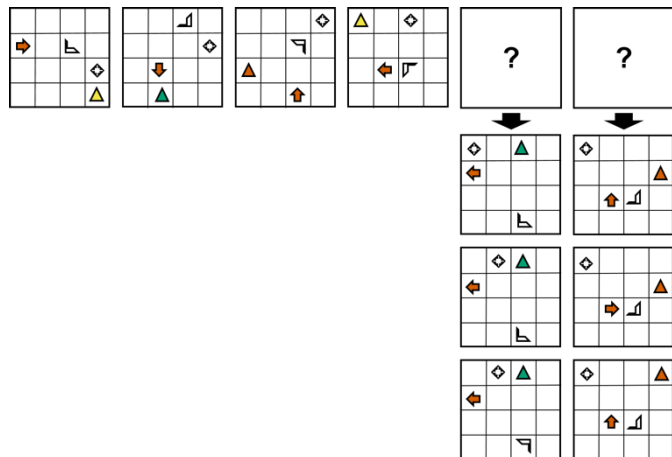
The symbol ➡ moves from its starting position diagonally downwards to the right until it bounces off the lower boundary and returns to the starting position in the same way (diagonally upwards to the left). Once there, it bounces off the left boundary and moves diagonally down to the right again. The symbol always turns x + 1 times to the right by 90 degrees. I.e. from matrix 1 to matrix 2 it rotates once by 90 degrees to the right. From matrix 2 to matrix 3 it rotates twice by 90 degrees to the right, and so on.

The symbol ⅂ moves vertically by one field in the third column and bounces off the upper or lower border. It rotates 90 degrees to the left from image to image.

The symbol ✛ moves along the outer borders counter clockwise one space at a time.

The symbol △ moves along the outer borders clockwise by two spaces at a time. In the process, it changes colour from yellow △ to green △ to orange △, etc.

Core Module	Mathematical Equations
Instructions	

In this task, you are supposed to solve systems of equations in such a way that all requirements are met. One system of equations always consists of several single equations.

Your task is to find out which numbers must be used for the unknowns (letters) in the equations so that all equations are correct.

There is always only one solution for each letter, in which all requirements are met.

Each letter can have be an integer between 1 and 20.

Example 1

A + 2 = **B**

B = 6

What number does **A** correspond to so that the equations are solved correctly?

Solution of Example 1

Because of the second equation, you know that **B** = 6. Replace **B** with the number 6 in the first equation and you get **A** + 2 = 6. Solve the first equation and you get **A** = 6 – 2 = 4. Therefore, the solution of the first example is **A** = 4. Any other solution is wrong.

Example 2

B = 2 × **A**

B + **A** = 12

What numbers do **A** and **B** correspond to so that the equations are solved correctly?

Solution of Example 2

The first equation defines that **B** = 2 × **A**. Putting this information in the second equation gives 2 × **A** + **A** = 12 or 3 × **A** = 12. Rearranging this equation gives **A** = 12 ÷ 3 = 4. Putting the number 4 into the first or second equation for **A** gives **B** = 8. Therefore, the solution of the second example is **A** = 4 and **B** = 8. Any other solution is wrong.

In the exam you have **25 minutes** to solve **20 systems of equations**. Please be as quick and accurate as possible. You are not allowed to take notes in the exam.

Core Module	Mathematical Equations
Exercises	

For the task type **Mathematical Equations**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

$$7 + A = 14$$
$$B - 3 = A$$

Exercise 2 – Difficulty: low

$$B \div 2 = A$$
$$B - A = 8$$

Exercise 3 – Difficulty: medium

$$3 \times C = A$$
$$A + C = 8$$
$$2 \times A + 2 \times C = B$$

Exercise 4 – Difficulty: medium

$$18 - B = A$$
$$3 \times A = C$$
$$B \div 2 = A$$

Exercise 5 – Difficulty: high

$$A - B + C - D = 2$$

$$10 \times B = C$$

$$5 \times B = A$$

$$11 + B = D$$

Exercise 6 – Difficulty: high

$$C + D - A = 1$$

$$5 \times C = D$$

$$13 - C = A$$

$$3 \times C - 1 = B$$

Solution – Exercise 1

7 + A = 14
B – 3 = A

A = 7
B = 10

The first equation makes it clear that A = 7 if you subtract 7 on both sides. If you insert this information into the second equation, you get B - 3 = 7. If you add 3 on both sides, you get the solution B = 10.

Solution – Exercise 2

B ÷ 2 = A
B – A = 8

A = 8
B = 16

Multiplying by 2 on both sides in the first equation gives B = 2A. Replacing the variable B in the second equation with this information gives 2A - A = 8. This means A = 8. Substituting the solution for A in the first equation gives B ÷ 2 = 8. Multiplying both sides by 2 gives B = 16.

Solution – Exercise 3

3 × C = A
A + C = 8
2 × A + 2 × C = B

A = 6
B = 16
C = 2

With the information from the first equation (3 × C = A or A = 3C), A can be replaced in the second equation so that it can be solved for C: 3C + C = 8 or 4C = 8. If you divide by 4 on both sides, you get C = 2. Thus, the solution of A can be calculated by substituting the value of C into the first equation: 3 × 2 = A. Therefore, A = 6. By substituting the solutions for A and C, the third equation can be solved for B: 2 × 6 + 2 × 2 = B. Therefore, B = 16.

Solution – Exercise 4

18 – B = A
3 × A = C
B ÷ 2 = A

A = 6
B = 12
C = 18

If you multiply by 2 on both sides in the third equation, you get B = 2A (alternatively, you can also continue the calculation with 0.5B = A, for example). If you replace B with this information in the first equation, you get 18 - 2A = A. If you add 2A on both sides, you get 18 = 3A. If you now divide by 3, you get A = 6. This information can be inserted into the third equation, so that you get B ÷ 2 = 6. If you multiply by 2 on both sides, you get B = 12. If you insert the result for A into the second equation, you get 3 × 6 = C, so C = 18.

Solution – Exercise 5

A – B + C – D = 2
10 × B = C
5 × B = A
11 + B = D

A = 5
B = 1
C = 10
D = 12

The information given in equations two, three and four for the variables A, B and C can be inserted into the first equation so that it can be solved for B: 5B - B + 10B - (11 + B) = 2. If you dissolve the bracket, you get 5B - B + 10 B - 11 - B = 2 or 13B - 11 = 2. If you add 11 on both sides, you get 13B = 13. If you divide by 13, you get the solution B = 1. This information can be inserted into the other equations and solved for the respective missing variable: 10 × 1 = C or C = 10, 5 × 1 = A or A = 5 and 11 + 1 = D or D = 12.

Solution – Exercise 6

C + D – A = 1
5 × C = D
13 – C = A
3 × C – 1 = B

A = 11
B = 5
C = 2
D = 10

The information given in equations two and three for the variables A and D can be inserted into the first equation, so that it can be solved for C: C + 5C - (13 - C) = 1. Dissolving the bracket gives C + 5C - 13 + C = 1 or 7C - 13 = 1. Adding 13 on both sides gives 7C = 14. Dividing by 7 gives the solution C = 2. This information can be inserted into the other equations and solved for the respective missing variable: 5 × 2 = D or D = 10, 13 - 2 = A or A = 11 and 3 × 2 - 1 = B or B = 5.

Core Module	Latin Squares
Instructions	

In this task you will see a 5x5 grid (a square containing 5 rows and 5 columns).

Some fields of the grid contain letters. Each letter can only appear once in each row and each column. Only the letters that are shown as response options (the row next to the grid) can appear in the grid.

Your task is to decide which letter belongs in the field with the question mark. Sometimes you need to fill in other fields in your mind before you can figure out what letter should replace the question mark.

If you know what the correct solution for the question mark field is, click on the correct response in the solution row.

	?					A
	A					B
	E					C
	D					D
C	B					E

Next, you will see two examples.

Example 1
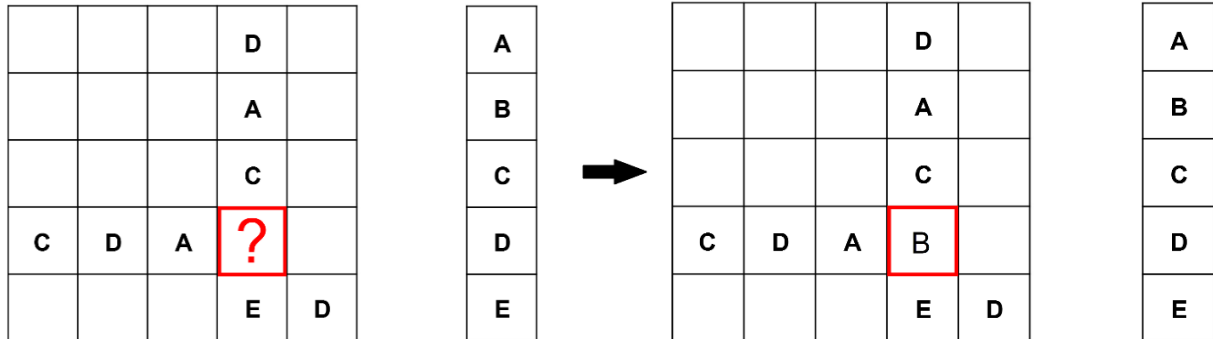
Solution of Example 1

In the first example, "B" needs to replace the red question mark, because all other letters D, A, C, and E already appear in this column.

Example 2
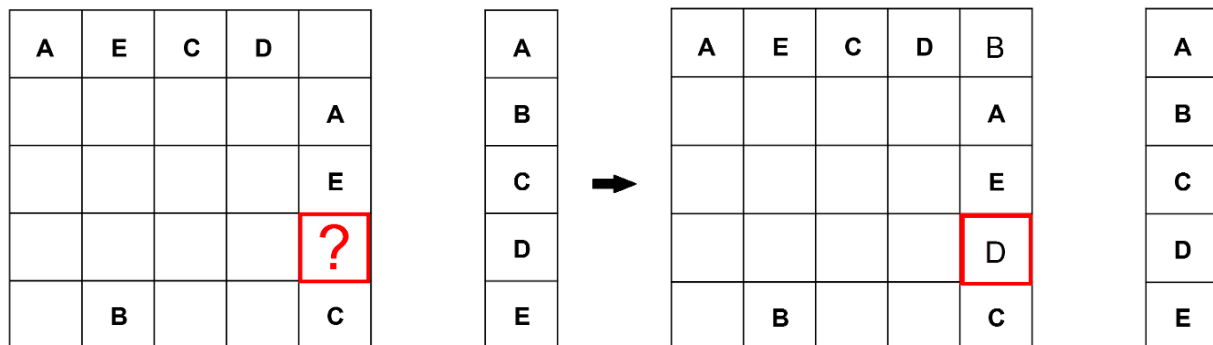
Solution of Example 1

In the second example, you first need to fill in "B" in the first row of the last column. "B" is the only letter, which does not already appear in this row and column. Then you can replace the question mark with "D", because it is the only letter that does not appear in the last column.

In the exam you have 25 minutes for 20 tasks. Please be as quick and accurate as possible! If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module	Latin Squares
Exercises – Solutions	

For the task type **Latin Squares**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Exercise 2 – Difficulty: low

		?		
			D	A
		E		D
A	D			B
D	B		C	

Exercise 3 – Difficulty: medium

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Exercise 4 – Difficulty: medium

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Exercise 5 – Difficulty: high

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Exercise 6 – Difficulty: high

?				C
	D	E	B	A
B		D	A	
	B	C		D

Note on the solution key

	α	β	γ	δ	ε
1	B	?	A	D	
2	A	B	E	C	
3		A			
4	C				
5	D	E		B	

Solution – Exercise 1

Solution = C

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Solution path:

- In column β, C and D are missing.
- C is already in row 4, so D must be inserted in β4.
- Consequently, C must be inserted in the place of the question mark.

Solution – Exercise 2

Solution = D

		?		
			D	A
		E		D
A	D			B
D	B		C	

Solution path:

- In the place of the question mark, D must be inserted because D is already given in all other columns and rows.

Solution – Exercise 3

Solution = B

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Solution path:

- In column γ, B and C are missing. At position γ4, only B can be inserted, since there is already a B in row 1. This also applies in γ4, or line 4 vice versa for C. Consequently, only a C can be inserted in γ1.
- A and D are missing in column β. A can only be in position β5, because A is already present in row 1. Consequently, only a D can be in position β1.
- From this follows that only one E can be inserted in ε1.
- In row 3 it is now noticeable that A can only be in position ε3, as it is already present in all columns and rows.

- Since a B still has to be inserted in column ε, and there is already a B in line 2, it can only be inserted at the position of the question mark.

Solution – Exercise 4

Solution = D

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Solution path:

- A and D are missing in the first row. A can only be inserted at position α1, since it is already in column γ. Consequently, D must be in position γ1.
- It is now noticeable that D is already present in four different rows and columns and can thus only be used in the place of the question mark.

Solution – Exercise 5

Solution = D

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Solution path:

- Only C can be inserted at position γ4.
- In row 3, A and D are missing. At position γ3, only an A can be inserted because it is already present in column α. Consequently, only a D can be inserted at position α1.
- Only A can be inserted at position β1.
- Only E can be inserted at position α1, as it is already present in all other rows and columns.
- Furthermore, C and B are missing in row 5, whereby only a C is inserted at position α5, since it is already present in column ε. Consequently, there is a B at position ε5.
- In row 1, D and B are still missing. Since B is already in column ε, B must be inserted at position γ1 and D at position ε1.
- At the position of the question mark, D must be inserted, since all other letters are already present in column γ.

Solution – Exercise 6

Solution = E

?				C
	D	E	B	A
B		D	A	
	B	C		D

Solution path:

- At position α3, only a C can be inserted, since all other letters are already in line 3.
- In row 5, A and E are missing. A must be in position α5 because it is already in column δ. Consequently, E is in position δ5.
- In row 4, C and E are missing. Only a C can be inserted at position β4, as it is already present in column ε. Consequently, there is an E at position ε4.
- At position ε2, only a B can be inserted, as all other letters in column ε are already present.
- In column γ, A and B are missing. At position γ1, only a B can be inserted, since there is already a B in the second row. Consequently, A must be inserted in γ2.
- In the first row, A, D and E must be inserted. A must be inserted in β1 because it is already present in all the other columns. Since E is already present in column δ, it must therefore be inserted in the position of the question mark.

Subject Module – Instructions and Exercises

Subject Module	Computer Science
General Instructions	

In this task type you see a text and a number of questions which you have to answer. There are 4 answer options for each question.

For each question, there is only one correct solution.

The text, the questions and the answer options may contain figures, tables and formulas.

For working on the entire subject test in the exam, you have 90 minutes in total. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

For practice and illustration of the subject module tasks, three exercises are available here.

Subject Module	Computer Science
Exercise 1	

Goals of computer security

Computer security is a branch of information technology, whose main goal is to protect systems and users from being compromised or attacked. In order to define precise protection mechanisms and attack countermeasures, computer security defines the following main security goals:

1. Integrity ensures that data cannot be modified.
2. Authenticity ensures that identity of the data issuer can be verified.
3. Confidentiality ensures that data are only visible by individuals with a key.
4. Availability ensures that data are guaranteed to be accessible to the users/systems when needed.

Question 1

One of the most famous attacks on TLS is the Heartbleed bug from 2014. The bug allowed an attacker to read random bytes from the server, which could potentially include usernames, passwords, and cryptographic material. What security goal does Heartbleed primarily violate?

a)   Confidentiality

b)   Integrity

c)   Authenticity

d)   Availability

Question 2

Some typical attacks affecting the security of currently used systems are different forms of ransomware. These attacks change the behaviour of the running system so that the system becomes unusable or its functionality differs. In addition to availability, what security goals does a ransomware primarily violate?

a)   Confidentiality

b)   Integrity

c)   Authenticity

d)   Nove of the above

Question 3

Single Sign-On is a typical mechanism to identify users in a system / in a set of systems. Assume there is an attacker who is able to bypass a Single Sign-On mechanism and log in as an arbitrary user. What security goal does such an attacker primarily violate?

a)   Confidentiality

b)   Integrity

c)   Authenticity

d)   Availability

Question 4

Denial-of-Service is a common security attack. With this attack, an adversary sends, for example, a large amount of data to a specific server. What security goal does a Denial-of-Service attack primarily violate?

a)   Confidentiality

b)   Integrity

c)   Authenticity

d)   Availability

Subject Module	Computer Science
Exercise 1 – Solutions	

Question 1

Solution: A

Heartbleed allowed any attackers to get in possession of usernames, passwords, and cryptographic material. Those data should only be visible to the server hosting the application. Because of this, Heartbleed targeted confidentiality.

Question 2

Solution: B

Ransomeware affects and modifies the running system and changes its behavior. Since the system gets modified, Ransomware primarily affects system itegrity.

Question 3

Solution: C

The primary goal of a Single Sign-On system is to ensure that users can authenticate to specific systems. Thus it primarily ensures authenticity. If an attacker is able to log in as a different user, the autenticity property is violated.

Question 4

Solution: D

Denial-of-Service attacks have the goal to make a system unavailable, by issuing a lot of requests to the target system. If the attack is successful, the server is not able to respond to requests sent by other parties and becomes unavailable. Thus, this attacks targets the availability goal.

Subject Module	Computer Science
Exercise 2	

Combinational Logic

Remember the boolean operation AND, OR and NOT. Their representation as logical gates in circuits is shown in Figure 1 below.
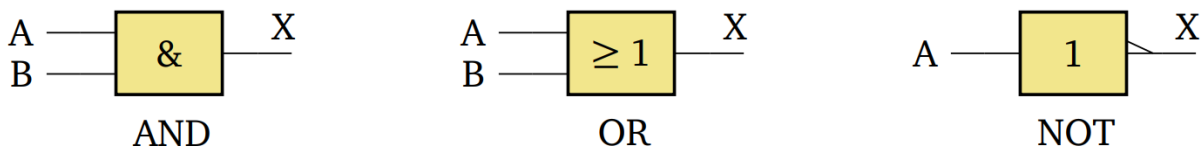
Figure 1. Logical gates in circuits of boolean operations

They are represented formally as functions $X = (A \wedge B)$ for AND, $X = (A \wedge B)$ for OR and $X = \overline{A}$ for NOT. The output X can be given by the combinations of the possible inputs. For boolean operations the input can only be 0 or 1 (sometimes also referred to as *false* and *true*). For completeness, the output of all three operations is presented using truth tables:

A	B	$A \wedge B$	$A \vee B$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

A	$\overline{A}$
0	1
1	0

Table 1. Truth table

These three boolean operations are the building blocks for more complex boolean functions or boolean circuits. As long as there are no cycles in the circuit, these circuits implement combinational logic. The resulting circuits are called combinatorial circuits.

From a truth table, we can immediately derive a boolean function or the combinatorial circuit by using what is called the canonical disjunctive normal form "(*CDNF*)". For each 1 that results from a function a *minterm* is created. A minterm is an AND operation over all inputs,

where 0-inputs are negated while 1-inputs are not. All minterms are finally combined with an OR operation.

As an example, the CDNF of AND derived from its truth table has only one minterm $X = (A \wedge B)$. It thus does not need to be used in an OR. However, the OR operation as CDNF has three minterms that need to be combined with OR: $X = (\overline{A} \wedge B) \vee (A \wedge \overline{B}) \vee (A \wedge B)$.

Question 1

Given the following truth table, which Boolean function does the table represent?

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

a) $X = \bar{A} \wedge \bar{B}$

b) $X = \overline{A \wedge B}$

c) $X = A \vee B$

d) $X = \bar{A} \vee B$

Question 2

Imagine a jewelry shop that has sensors at all of its windows, doors, and showcases. Its burglar alarm should go off if at least one of the sensors detects unusual conditions. Which Boolean operation best describes the behavior of the alarm system?

a)   AND operation

b)   OR operation

c)   one AND combined with several OR operations

d)   one OR combined with several AND operations

Question 3

The following combinatorial circuit has three inputs A, B and C The corresponding truth table has some missing entries. What are these missing values?
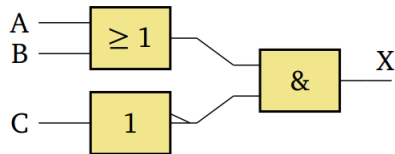
A	B	C	X
0	0	0	0
0	0	1	{i}
0	1	0	1
0	1	1	{ii}
1	0	0	{iii}
1	0	1	0
1	1	0	1
1	1	1	0

a) $i = 0; ii = 1; iii = 1$

b) $i = 1; ii = 1; iii = 1$

c) $i = 0; ii = 0; iii = 0$

d) $i = 0; ii = 0; iii = 1$

Question 4

What is the correct boolean function for the shown combinatorial circuit?
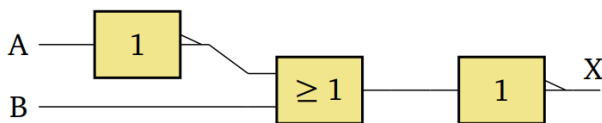
a) $X = \overline{\bar{A} \vee B}$

b) $X = \overline{A \vee \bar{B}}$

c) $X = A \wedge \overline{B}$

d) $X = \overline{\bar{A} \wedge B}$

Question 5

What is the CDNF for the following truth table?

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

a) $X = (\bar{A} \wedge B) \vee (A \wedge \bar{B}) \vee (A \wedge B)$

b) $X = (\bar{A} \wedge \bar{B}) \vee (A \wedge B)$

c) $X = (\bar{A} \wedge B)$

d) $X = (\bar{A} \wedge B) \vee (A \wedge \bar{B})$

Question 6

You have a truth table with four inputs A, B, C and D. In one of the lines the result X is 1 for the following combination of inputs:

A	B	C	D	X
1	0	1	1	1

What is the correct minterm for this line?

a) $A \wedge \bar{B} \wedge C \wedge D$

b) $A \vee \bar{B} \vee C \vee D$

c) $\bar{A} \wedge B \wedge \bar{C} \wedge \bar{D}$

d) $A \wedge C \wedge D$

Question 7

What is the truth table for the following circuit?:
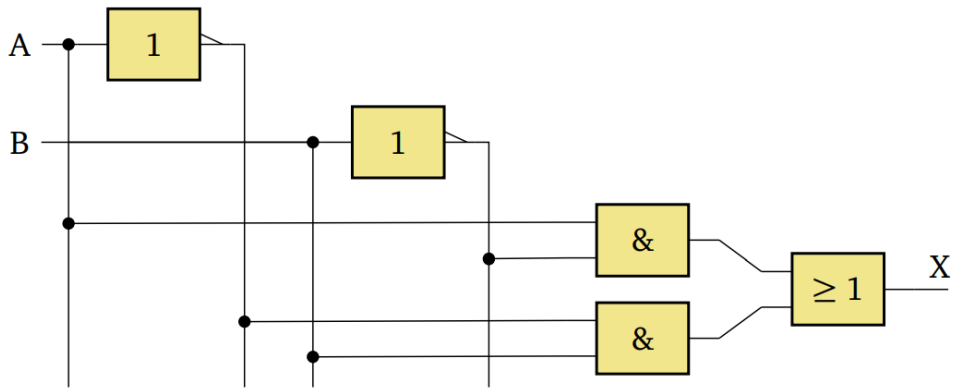
a)

A	B	X
0	0	1
0	1	1
1	0	0
1	1	0

b)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

c)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

d)

A	B	X
0	0	0
0	1	0
1	0	1
1	1	1

Subject Module	Computer Science
Exercise 2 – Solutions	

Question 1

Solution: A

We have to look at the assignments of A and B that lead to X being evaluated to 1. There is only one assignment that evaluates to $X = 1$. Therefore, X is only 1 if the negation of A is 1 and the negation of B is 1.

Question 2

Solution: B

Each of the sensor can be represented with one variable that is either 1 (sensor detects anomaly) or 0 (sensor detects no anomaly). Since only one of the sensors needs to detect an anomaly for the alarm to go off (entire Boolean formula is 1), we put all the variables in one large OR operation.

Question 3

Solution: D

The corresponding formula for the circuit is $X = (A \lor B) \land \bar{C}$. We can just plug in the assignments to get X. For {i} and {ii} C is not 0. Only {iii} evaluates to 1 since both $\bar{C}$ and A are 1.

Question 4

Solution: A

We go from left to the right to derive the formula and determine the order of operations. A is negated and used as input, which is $\bar{A}$ as formula. Now we concatenate the result with B using an OR operation, which is equivalent to $\bar{A} \vee B$. Finally, the entire result X is negated, which means: $X = \overline{\bar{A} \vee B}$.

Question 5

Solution: D

We use the approach from the testlet stem to derive a CDNF and create minterms for the second and third row, where X = 1. These are $\overline{\bar{A} \wedge B}$ and $\overline{A \wedge \bar{B}}$. Now we just combine these minterms with A to arrive at the correct result $X = \left(\bar{A} \wedge B \right) \vee \left(A \wedge \bar{B} \right)$.

Question 6

Solution: A

Since in the exercise a minterm is explicitly mentioned, we have an AND operation for this line. So, we simply negate the variables that are 0 in the table (B) and combine all of the literals using an AND operation.

Question 7

Solution: C

The two AND operations are $\left(A \wedge \bar{B} \right)$ for the upmost gate and $\left(\overline{\bar{A} \wedge B} \right)$ for the lower one. They are combined using an OR operation, so the resulting CDNF is: $\left(A \wedge \bar{B} \right) \vee \left(\overline{\bar{A} \wedge B} \right)$. Now we can just check which given truth table satisfies this CDNF.

Subject Module	Computer Science
Exercise 3	

Linear Transformations of High-Dimensional Data Sets

Linear transformations are a fundamental concept in data science. They are often used to reduce the dimensions of data sets (commonly represented as matrices). This reduction is necessary to make high-dimensional data easier to process and analyse. The basis for such transformations is *linear matrix algebra*. Fundamental concepts in matrix algebra, which are also the basis for the application of linear transformations, are the *rank*, the *dimensions*, and the *eigenvalues* of a matrix.

The rank of a matrix is its number of linearly independent rows or columns. A row or column is linearly independent if it cannot be expressed as a linear combination of the other rows or columns in the matrix. The rank of a matrix can be found using row or column operations to reduce the matrix to row-echelon form, and then counting the number of non-zero rows or columns.

The dimension of a matrix is given by its number of rows and columns. For example, a matrix with *m* rows and *n* columns has a dimension of $m \, x \, n$.

An eigenvalue is a scalar value that represents how the linear transformation stretches or shrinks a vector in a given direction. To calculate the eigenvalues of a matrix, solve the equation:

$$\det(A - \lambda I) = 0$$

where *A* is the matrix, *λ* is the eigenvalue, and *I* is the identity matrix of the same size as *A*. The *determinant det* of a matrix is a scalar value that indicates whether the matrix is invertible and describes the scaling factor of the linear transformation represented by the matrix. For a *2x2* matrix, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant is calculated as:

det(A) = ad – bc.

The resulting equation is a polynomial of degree *n*, where n is the size of the matrix. The roots of this polynomial are the eigenvalues of the matrix. Once we have found the eigenvalues, we can then find the corresponding eigenvectors by solving the equation:

$$(A - \lambda I) * x = 0$$

where *x* is the eigenvector corresponding to the eigenvalue *λ*.

Principal Component Analysis (*PCA*) is a common linear transformation method that is used to reduce the dimensionality of the data while retaining as much of the variation in the data as possible. The resulting lower-dimensional space is called the principal subspace. PCA can be used to identify patterns in data and to identify the most important variables for further analysis.

Another technique for linear transformation is *Linear Discriminant Analysis* (*LDA*). LDA is a supervised learning method that is used to find linear combinations of features that can best separate different classes in the data. The resulting lower-dimensional space is optimized for classification performance. LDA can be used to identify the variables that are most important for predicting a specific outcome.

Reconstruction error is a measure of how well a reduced-dimensionality representation of a data set can reconstruct the original data points. It is defined as the sum of the squared distances between each original data point and its reconstructed counterpart in the reduced-dimensionality space. In other words, it is a measure of how much information is lost when reducing the dimensionality of the data.

Question 1

Please consider the following matrix:

| 2 1 |

| 1 2 |

What is the rank of the matrix?

a)   1

b)   2

c)   4

d)   3

Question 2

Please consider the following matrix:

| 2 1 |

| 1 2 |

What are the eigenvalues of the matrix?

a)   2, 2

b)   1, 3

c)   1, 1

d)   3, 1

Question 3

Please consider the following matrix:

| 2 1 |

| 1 2 |

If the matrix is transformed using PCA, what is the dimensionality of the resulting matrix when keeping only the first principal component?

a) 2×2

b) 1×1

c) 1×2

d) 2×1

Question 4

Please consider the following matrix:

| 2 1 |

| 1 2 |

If the matrix is transformed using LDA with two classes, what is the dimensionality of the resulting matrix when keeping only the first linear discriminant?

a) 2×1

b) 1×1

c) 1×2

d) 2×2

Question 5

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

What is the dot product of u and v?

a)   12

b)   6

c)   32

d)   56

Question 6

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following statements is true about a matrix $A \in R\, n \times n$ with n-dimensional column vectors a_i, if A is invertible?

a)   A has to have full rank, which is equivalent to $a_1, \ldots, a_n$ being linearly independent and the determinant must be non-zero.

b) $a_1,\ldots, a_n$ are linearly dependent, and must <u>not</u> have a full rank.

c)   The determinant of A has to be exactly zero, and $a_1, \ldots, a_n$ are linearly independent.

d)   A has to have full rank. The determinant of A can be zero or non-zero.

Question 7

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following is not a commonly used linear method for dimensionality reduction?

a)   They cannot handle missing values in the data set.

b)   They are computationally expensive.

c)   They are prone to overfitting.

d)   They can only handle linear relationships between variables.

Question 8

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following is a common approach to select the number of principal components to retain in PCA?

a)   Retain principal components that explain at least 90% of the variance in the data set.

b)   Retain all principal components.

c)   Retain only the first principal component.

d)   Retain principal components that explain the least amount of variance in the data set.

Subject Module	Computer Science
Exercise 3 – Solutions	

Question 1

Solution: B

The matrix is full rank, which can be determined by calculating the determinant $\det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 2 * 2 - 1 * 1 = 3$, which is not 0. In a full rank matrix, all rows are independent, and the rank is equal to the largest possible rank, which is 2 for this matrix.

Question 2

Solution: B

We can find the eigenvalues by solving the characteristic equation $\det(A - \lambda I) = \det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \det\begin{pmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{pmatrix} = (2 - \lambda)(2 - \lambda) - 1 * 1 = 0$

The values that solve this equation are 1 and 3.

Question 3

Solution: D

It is a general convention in data science that rows represent individual observations, while columns represent the features. The first principal component determines the direction of the maximum variance for all data points. If we apply it to the given matrix, we reduce the feature space to 1. The resulting matrix therefore has a dimensionality of $2\ x\ 1$, if we just keep the first principal component.

Question 4

Solution: A

We simply calculate the sum of the products of their corresponding components: $[1,2,3] * [4,5,6] = 1 * 4 + 2 * 5 + 3 * 6$, which is 32.

Question 5

Solution: C

If we view a matrix A with size $m \, x \, n$ as a linear transformation $T: R^n \to R^m$, with $T(x) = Ax$, the rows must be independent for the transformation to be reversible (*T* has to be bijective). If the rows are not independent, two different vectors *x* and *y* could potentially map to the same vector in R^m, which would violate the bijective property of a reversible transformation. In other words, if the rows are not independent, information is lost in the transformation, which cannot be recovered.

Question 6

Solution: A

The Random Forest Algorithm is a machine learning method for classification and regression by building decision trees and aggregating them. It is not commonly used for dimensionality reduction, since there are no transformations to the feature space directly.

Question 7

Solution: D

Linear methods for dimensionality reduction operate under the assumption that the variables have linear relationships. This, however, is not always the case, which is especially true for real-world data.

Question 8

Solution: A

In PCA, each principal component explains some of the total variance of the dataset. The first principal component captures the most variance, the second captures the second most, and so on. By using the variance as an approximated measure of information, we aim to retain 90%, while reducing the dimensionality as far as possible.

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