dMAT – Digital Master Assessment Test

Subject Modules

Battery Science and Technology in Engineering

Preparatory Materials for Test Takers

dMAT
Digitaler Mastertest

Imprint

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Instructions for the use of the preparation materials for test takers

Dear dMAT participant,

these preparation materials will help you to prepare well for the dMAT exam. Here you get

- general information on the content and structure of the test,
- detailed instructions and hints on how to work through the different task types as well as
- the possibility to work on exercises for each task type in different levels of difficulty including sample solutions.

Read all the information carefully to become well acquainted with the dMAT. The preparation materials are primarily intended to help you to prepare for the exam in terms of content. You can get hints and examples on the digital form of the test in information videos on www.d-mat.de.

Note

The detailed instructions for the individual task types are only available in these preparation materials! In the dMAT exam you will only see short explanations of the processing as a reminder.

We wish you lots of success!

Your dMAT team

General information about the dMAT

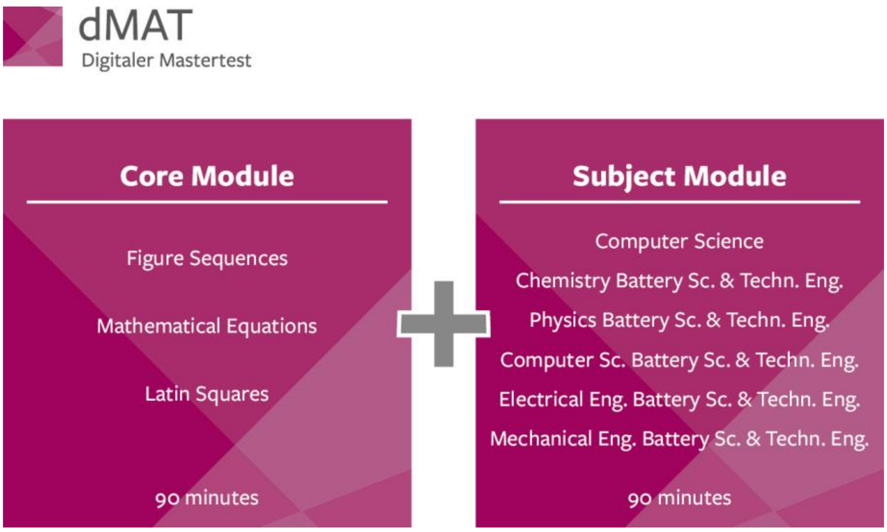
The digital Master Assessment Test (dMAT) is a new study aptitude test used for the admission of international applicants to Master's degree programmes in Germany. Taking the test allows a fair comparison of applicants across different national education systems.

The dMAT is offered digitally. Developed in close professional collaboration with German universities, the test accurately reflects the requirements of the intended degree programmes and is thus a valid instrument for assessing study ability for master's degree programmes.

The dMAT exam are evaluated centrally at the TestDaF Institute in Bochum. The test is standardized, which ensures that all participants can be compared with each other. In addition, the format of the test is based on the internationally recognized standards for psychodiagnostic tests, the test is reliable and objective.

Structure of the dMAT

The dMAT consists of two parts: A Core Module that tests general study aptitude, and Subject Modules that test subject-specific aptitude as well as the ability to apply the knowledge acquired in the course of your Bachelor's studies. The following graphic illustrates the structure of the test and the current subject modules.



The duration of the exam itself is about three hours with a break of 30 minutes between the two parts of the exam.

The **Core Module** measuring general study ability consists of three subtests that measure general cognitive abilities relevant to a master's degree programme in Germany. To a certain extent, the core module allows participants to be compared across the respective subject modules.

The production of the **Subject Modules** is based on extensive scientific studies by experts, so that the exam content is representative of the respective fields of study. The examination tasks are knowledge-based and consist of a combination of a typical subject-related problem (input) and corresponding single-choice questions. The dMAT requires subject knowledge based on your Bachelor's studies and application skills. You can familiarize yourself with the exact requirements and instructions of the individual task types in the next sections.

Only for the five Subject Modules for the MSc in Battery Science and Technology in Engineering there are tasks on two different levels of difficulty:

- **Basic tasks** in all relevant fields that is, Chemistry, Physics, Computer Science, Electrical Engineering and Mechanical Engineering.
- **Advanced tasks** that require Bachelor-level knowledge and skills only in the chosen field (i.e., Chemistry **or** Physics **or** Computer Science **or** Electrical Engineering **or** Mechanical Engineering)

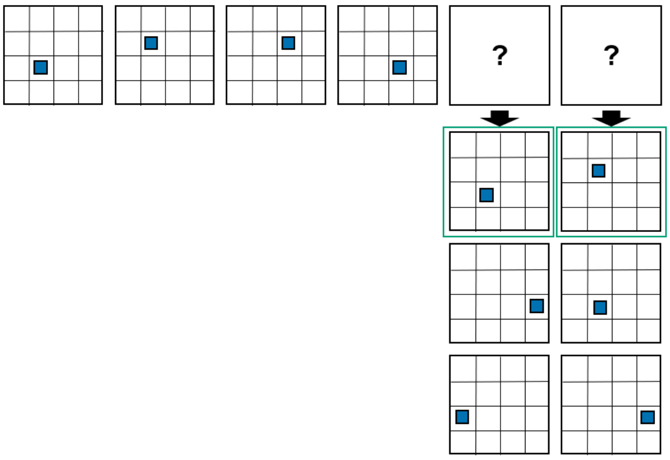
Please note: You may not take notes throughout the exam.

Core Module – Instructions and Exercises

Core Module	Figure Sequences
Instructions	

In this task you will see a series of pictures (matrices). The figures in the matrices can change their **position**, **colour**, and/or **orientation** from one matrix to the next according to specific rules. It is your task to continue the series logically and to determine what the next two matrices look like.

Example Task

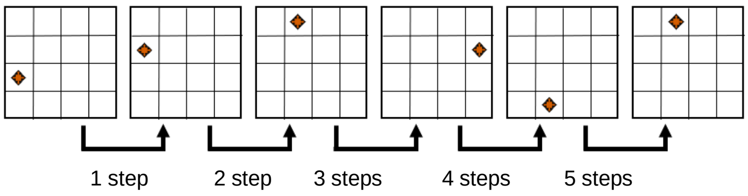


Solution

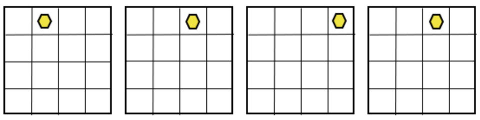
The blue square always moves one field clockwise within the four middle fields. Therefore, for the fifth matrix the first response option is correct, and for the sixth matrix as well.

Rules

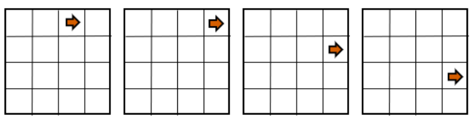
- Figures can change their colour.
- Figures can rotate around their own axis.
- Figures can move in the matrix. Vertical, horizontal, and diagonal movements are allowed. Figures cannot change from one diagonal movement to another type of movement.
- Figures can also change their movement, colour or orientation by x + 1. Example: If a figure moves one step from matrix 1 to matrix 2, it moves 2 steps from matrix 2 to matrix 3, then 3 steps, etc.



- Figures cannot disappear or overlap.
- Figures cannot leave the matrix. If they come up against an outer boundary, they can EITHER
- bounce off OR



- move along the outer boundary.

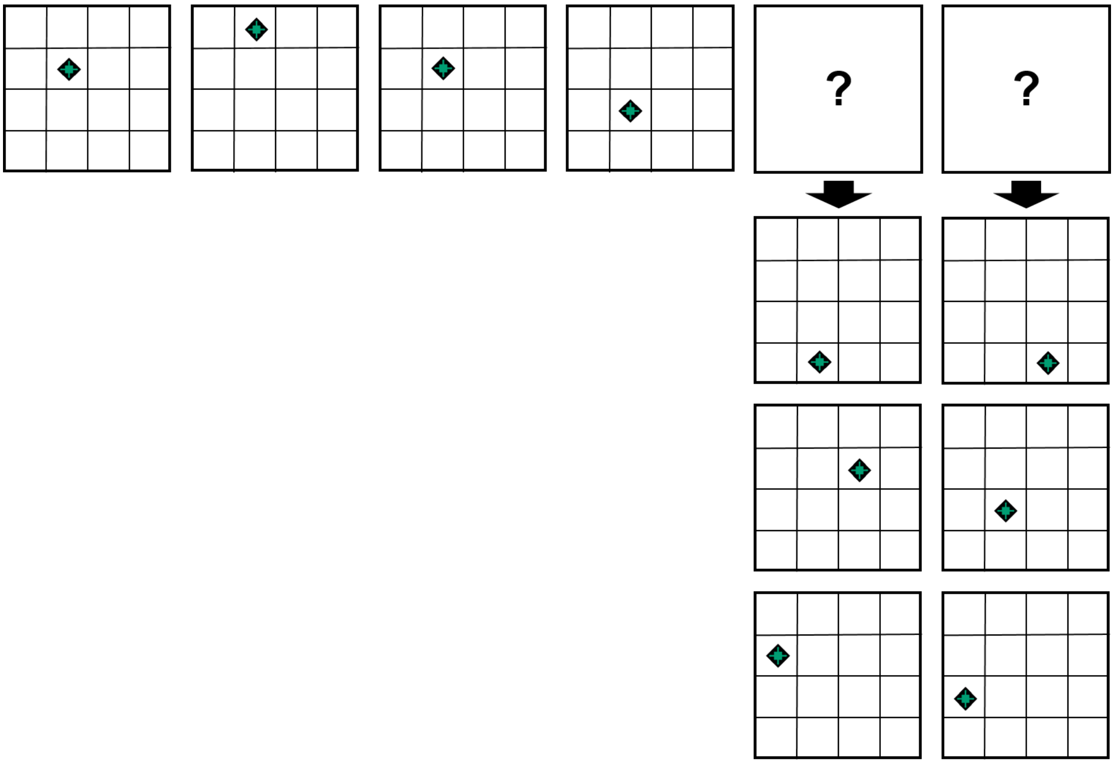


In the exam you have a total of **25 minutes** for **20** series of matrices. Please be as quick and accurate as possible. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

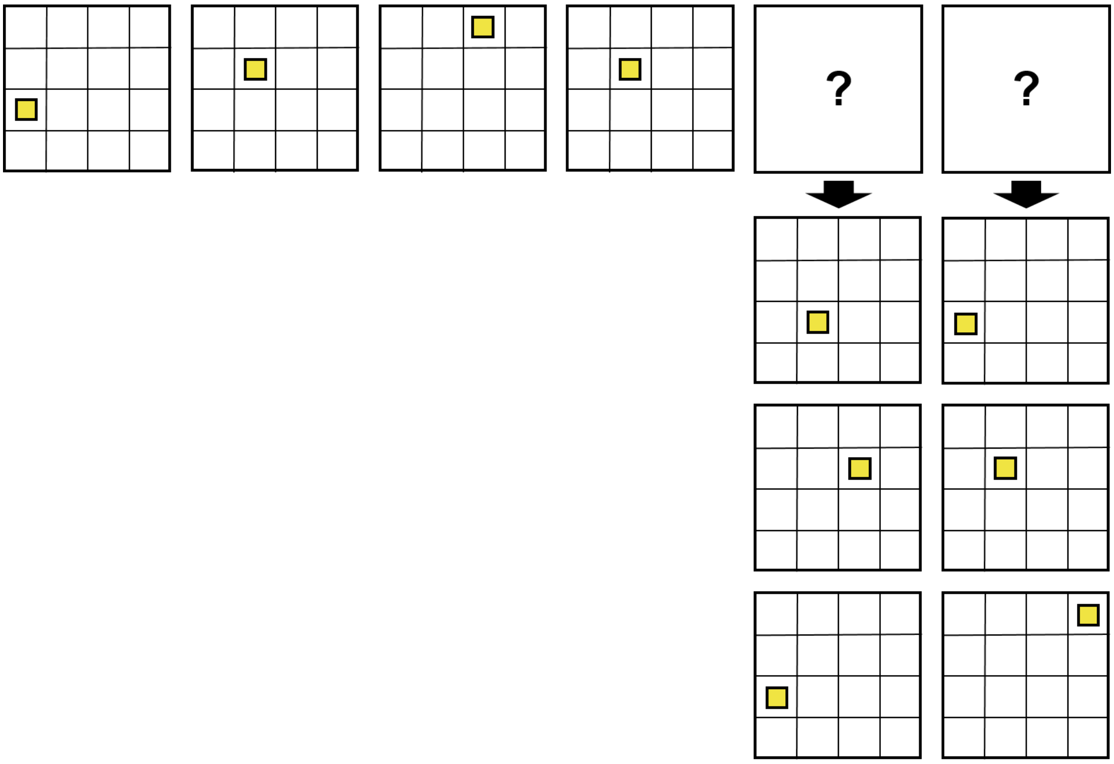
Core Module	Figure Sequences
Exercises	

For the task type **Figure Sequences** there are six exercises available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

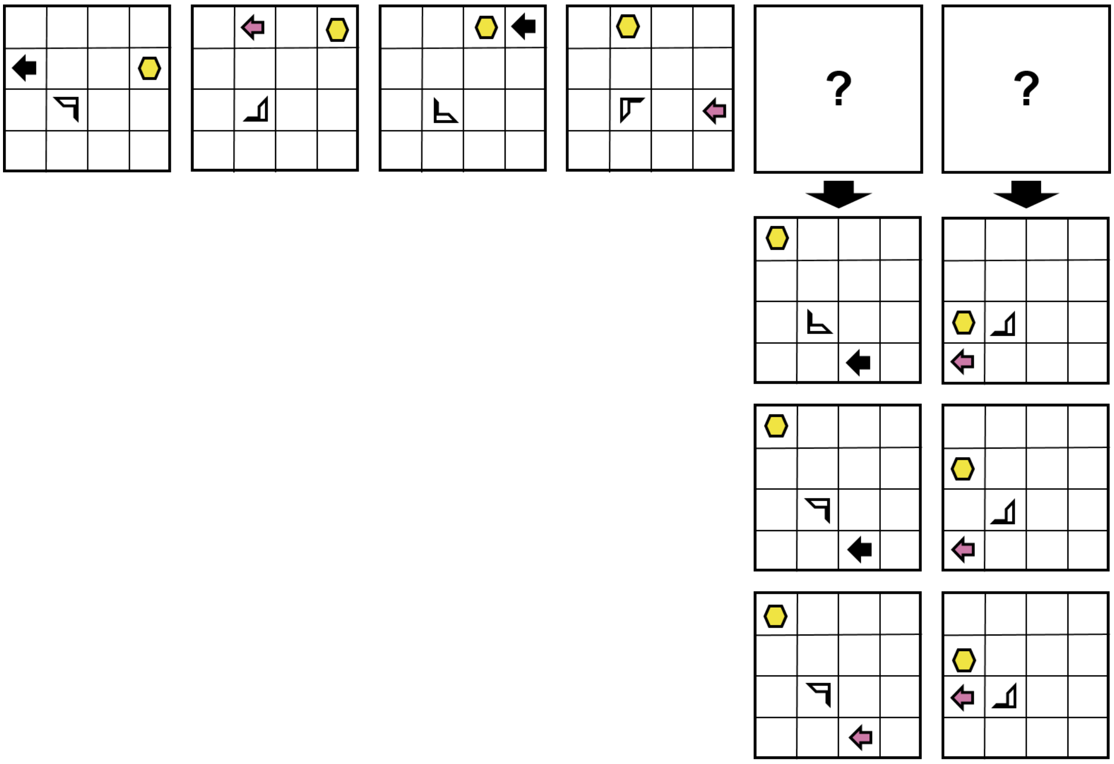
Exercise 1 – Difficulty: low



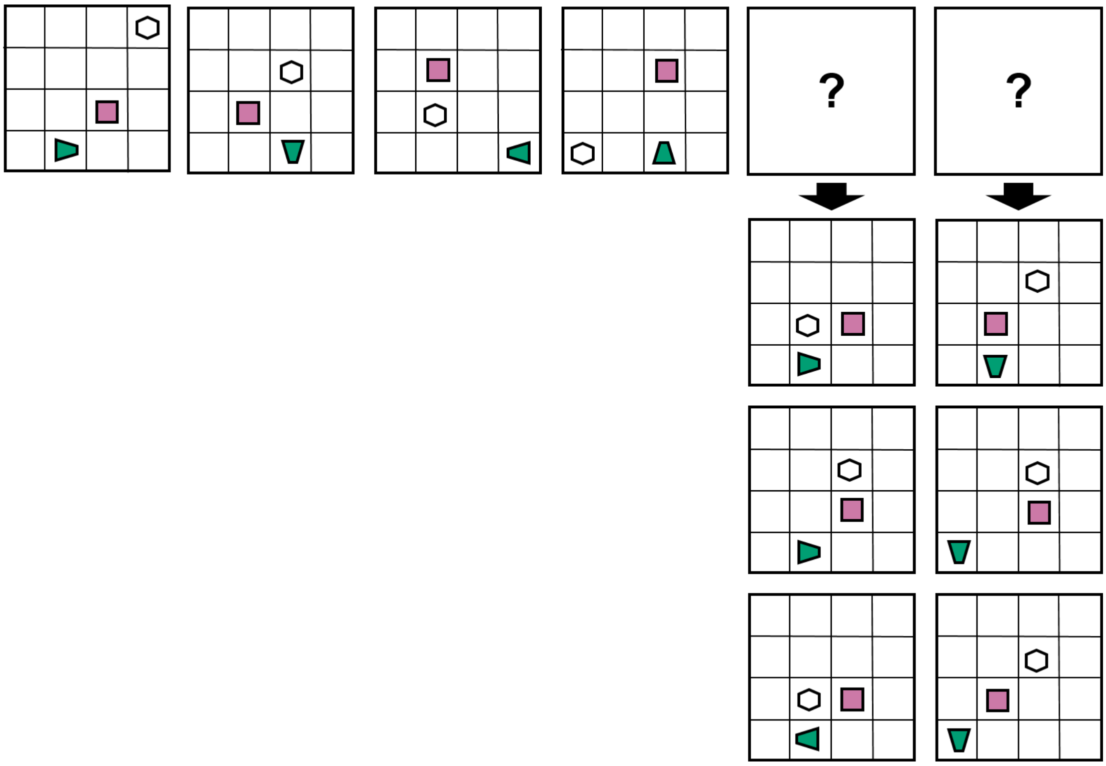
Exercise 2 – Difficulty: low



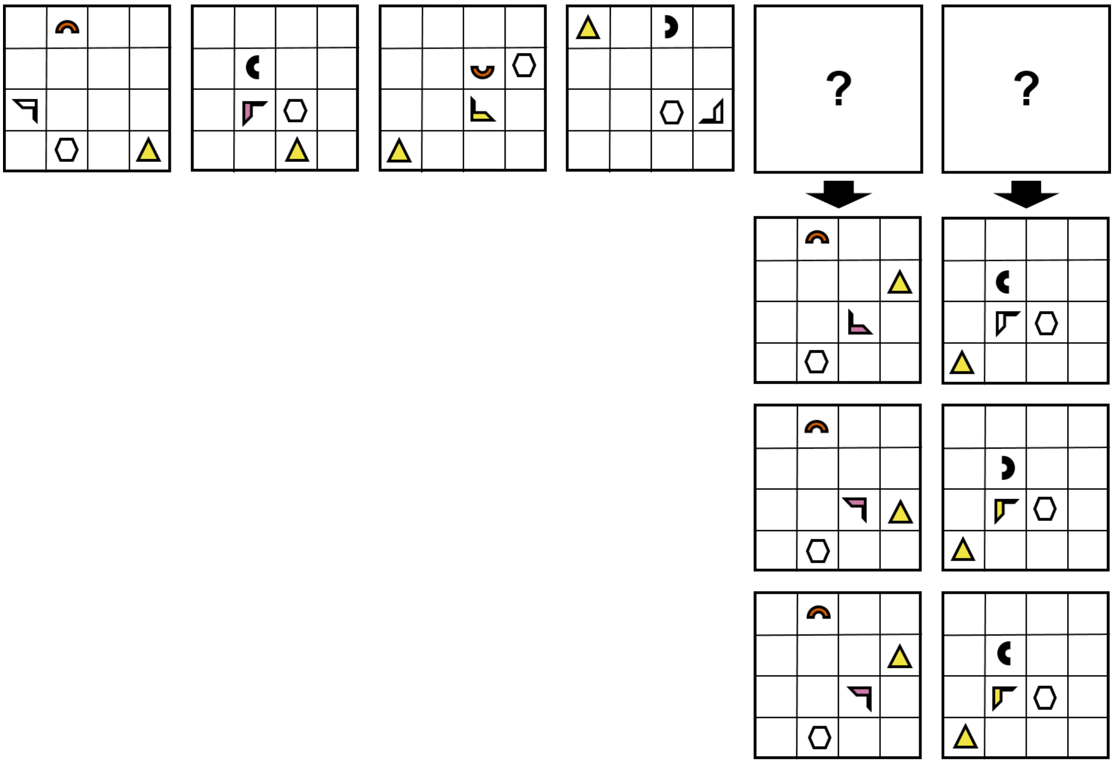
Exercise 3 – Difficulty: medium



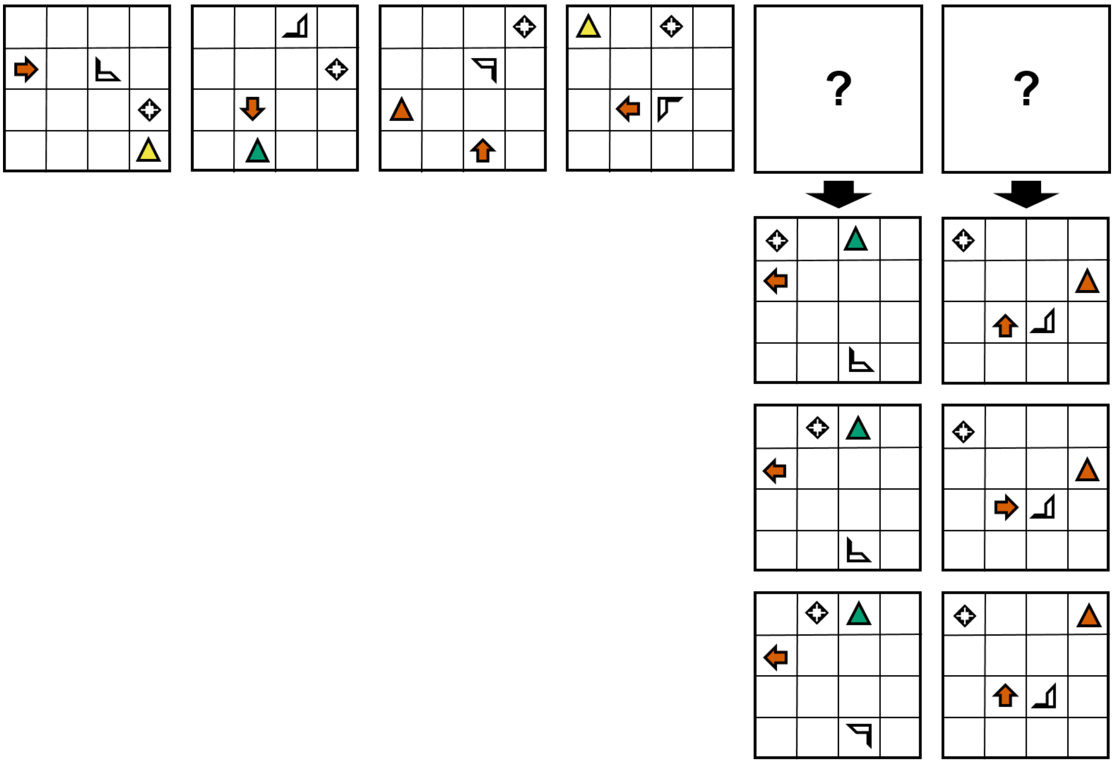
Exercise 4 – Difficulty: medium



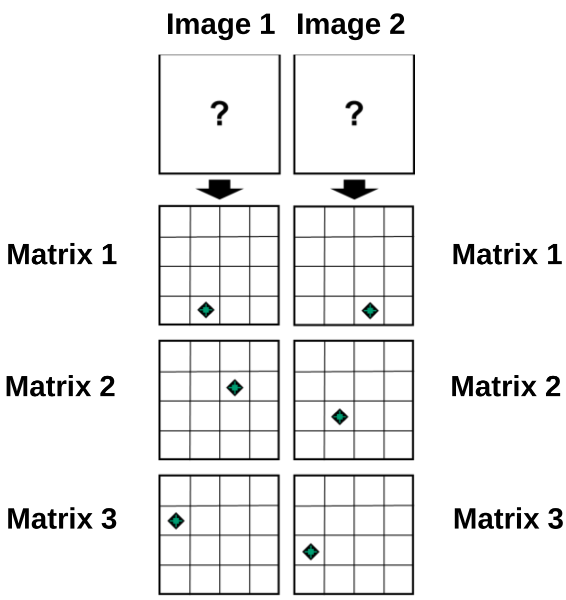
Exercise 5 – Difficulty: high



Exercise 6 – Difficulty: high



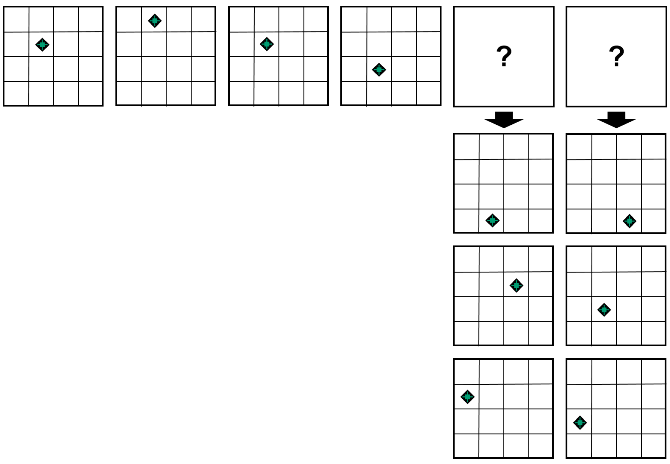
Note on the solution key



Solution – Exercise 1

Image 1: Matrix 1

Image 2: Matrix 2

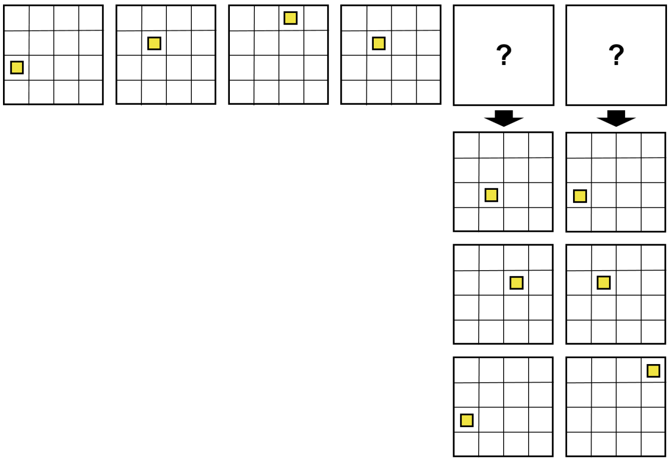


The symbol  moves vertically one field at a time in the second column and bounces off the upper or lower boundary.

Solution – Exercise 2

Image 1: Matrix 3

Image 2: Matrix 2

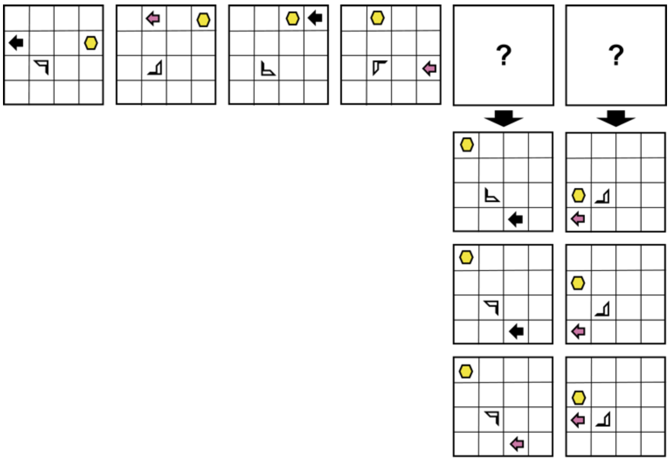


The symbol  always moves one space diagonally upwards to the right from its starting position until it bounces off the upper boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 3

Image 1: Matrix 2

Image 2: Matrix 2



The symbol  moves along the outer borders clockwise by two squares at a time. It changes its colour alternately from black  to pink .

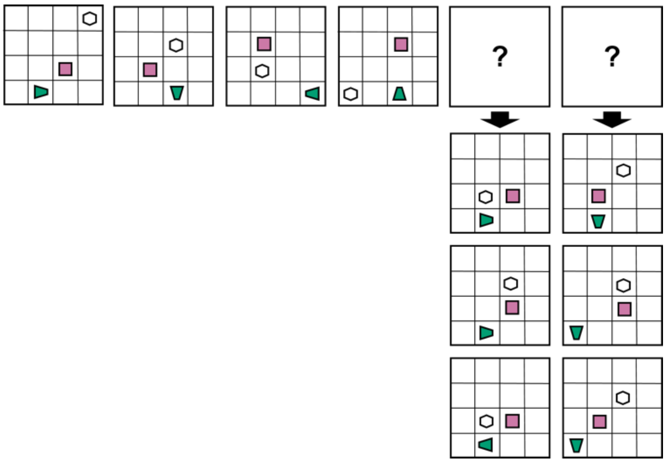
The symbol  rotates 90 degrees to the right from image to image.

The symbol  moves along the outer borders counter clockwise one space at a time.

Solution – Exercise 4

Image 1: Matrix 1

Image 2: Matrix 3



The symbol moves horizontally by one field in the fourth row and bounces off the right or left border. It rotates 90 degrees to the right from image to image.

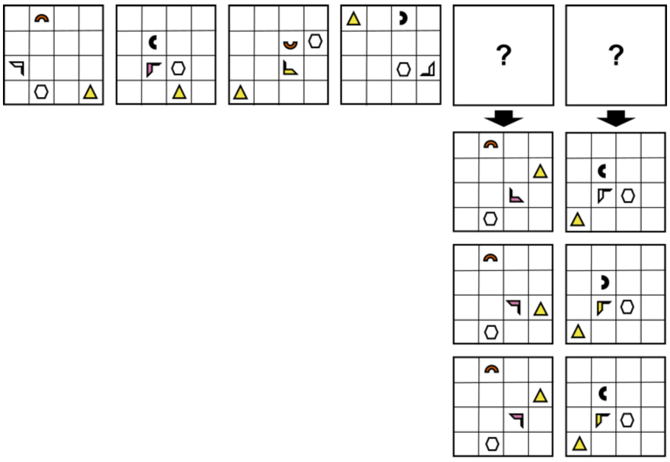
The symbol moves from its starting position one field at a time from image to image. The order of the directions in which the symbol moves is: left, up, right, down, and so on.

The symbol moves from its starting position diagonally downwards to the left until it bounces off the bottom left corner and returns the same way to the top right corner (diagonally upwards to the right).

Solution – Exercise 5

Image 1: Matrix 3

Image 2: Matrix 3



The symbol moves along the outer borders clockwise by x + 1 fields (i.e. from matrix 1 to matrix 2 one field, from matrix 2 to matrix 3 two fields, and so on).

The symbol moves horizontally by one field in the third row and bounces off the right or left border. In doing so, it turns 90 degrees to the left from image to image and changes its colour from white to pink to yellow , and so on.

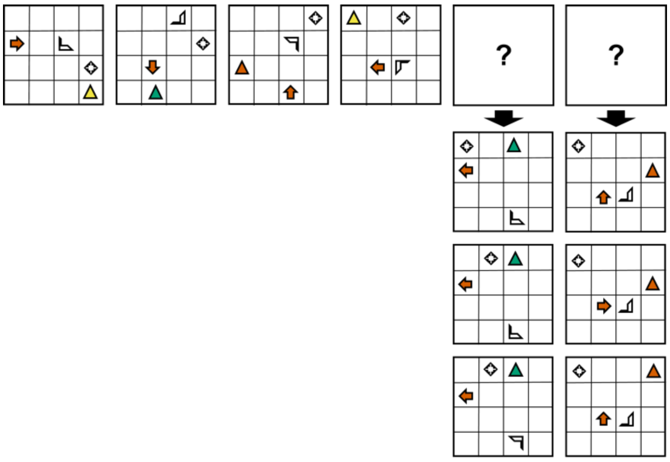
The symbol moves from its starting position one field at a time from image to image. The sequence of directions in which the symbol moves is: down, right, up, left, and so on. It turns 90 degrees to the left and changes its colour alternately from orange to black.

The symbol moves diagonally upwards to the right from its starting position until it bounces off the right boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 6

Image 1: Matrix 2

Image 2: Matrix 1



The symbol moves from its starting position diagonally downwards to the right until it bounces off the lower boundary and returns to the starting position in the same way (diagonally upwards to the left). Once there, it bounces off the left boundary and moves diagonally down to the right again. The symbol always turns x + 1 times to the right by 90 degrees. I.e. from matrix 1 to matrix 2 it rotates once by 90 degrees to the right. From matrix 2 to matrix 3 it rotates twice by 90 degrees to the right, and so on.

The symbol moves vertically by one field in the third column and bounces off the upper or lower border. It rotates 90 degrees to the left from image to image.

The symbol moves along the outer borders counter clockwise one space at a time.

The symbol moves along the outer borders clockwise by two spaces at a time. In the process, it changes colour from yellow to green to orange , etc.

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Core Module	Mathematical Equations
Instructions	

In this task, you are supposed to solve systems of equations in such a way that all requirements are met. One system of equations always consists of several single equations.

Your task is to find out which numbers must be used for the unknowns (letters) in the equations so that all equations are correct.

There is always only one solution for each letter, in which all requirements are met.

Each letter can have be an integer between 1 and 20.

Example 1

A + 2 = **B**

B = 6

What number does **A** correspond to so that the equations are solved correctly?

Solution of Example 1

Because of the second equation, you know that **B** = 6. Replace **B** with the number 6 in the first equation and you get **A** + 2 = 6. Solve the first equation and you get **A** = 6 – 2 = 4. Therefore, the solution of the first example is **A** = 4. Any other solution is wrong.

Example 2

B = 2 × **A**

B + **A** = 12

What numbers do **A** and **B** correspond to so that the equations are solved correctly?

Solution of Example 2

The first equation defines that **B** = 2 × **A**. Putting this information in the second equation gives 2 × **A** + **A** = 12 or 3 × **A** = 12. Rearranging this equation gives **A** = 12 ÷ 3 = 4. Putting the number 4 into the first or second equation for **A** gives **B** = 8. Therefore, the solution of the second example is **A** = 4 and **B** = 8. Any other solution is wrong.

In the exam you have **25 minutes** to solve **20 systems of equations**. Please be as quick and accurate as possible. You are not allowed to take notes in the exam.

For the task type **Mathematical Equations**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

$7 + A = 14$

$B - 3 = A$

Exercise 2 – Difficulty: low

$B \div 2 = A$

$B - A = 8$

Exercise 3 – Difficulty: medium

$3 \times C = A$

$A + C = 8$

$2 \times A + 2 \times C = B$

Exercise 4 – Difficulty: medium

$18 - B = A$

$3 \times A = C$

$B \div 2 = A$

Exercise 5 – Difficulty: high

$A - B + C - D = 2$

$10 \times B = C$

$5 \times B = A$

$11 + B = D$

Exercise 6 – Difficulty: high

$C + D - A = 1$

$5 \times C = D$

$13 - C = A$

$3 \times C - 1 = B$

Solution – Exercise 1

$7 + A = 14$
$B - 3 = A$

$A = 7$
$B = 10$

The first equation makes it clear that $A = 7$ if you subtract 7 on both sides. If you insert this information into the second equation, you get $B - 3 = 7$. If you add 3 on both sides, you get the solution $B = 10$.

Solution – Exercise 2

$B \div 2 = A$
$B - A = 8$

$A = 8$
$B = 16$

Multiplying by 2 on both sides in the first equation gives $B = 2A$. Replacing the variable B in the second equation with this information gives $2A - A = 8$. This means $A = 8$. Substituting the solution for A in the first equation gives $B \div 2 = 8$. Multiplying both sides by 2 gives $B = 16$.

Solution – Exercise 3

$3 \times C = A$
$A + C = 8$
$2 \times A + 2 \times C = B$

$A = 6$
$B = 16$
$C = 2$

With the information from the first equation ($3 \times C = A$ or $A = 3C$), A can be replaced in the second equation so that it can be solved for C: $3C + C = 8$ or $4C = 8$. If you divide by 4 on both sides, you get $C = 2$. Thus, the solution of A can be calculated by substituting the value of C into the first equation: $3 \times 2 = A$. Therefore, $A = 6$. By substituting the solutions for A and C, the third equation can be solved for B: $2 \times 6 + 2 \times 2 = B$. Therefore, $B = 16$.

Solution – Exercise 4

$18 - B = A$
$3 \times A = C$
$B \div 2 = A$

$A = 6$
$B = 12$
$C = 18$

If you multiply by 2 on both sides in the third equation, you get $B = 2A$ (alternatively, you can also continue the calculation with $0.5B = A$, for example). If you replace B with this information in the first equation, you get $18 - 2A = A$. If you add 2A on both sides, you get $18 = 3A$. If you now divide by 3, you get $A = 6$. This information can be inserted into the third equation, so that you get $B \div 2 = 6$. If you multiply by 2 on both sides, you get $B = 12$. If you insert the result for A into the second equation, you get $3 \times 6 = C$, so $C = 18$.

Solution – Exercise 5

$A - B + C - D = 2$
$10 \times B = C$
$5 \times B = A$
$11 + B = D$

$A = 5$
$B = 1$
$C = 10$
$D = 12$

The information given in equations two, three and four for the variables A, B and C can be inserted into the first equation so that it can be solved for B: $5B - B + 10B - (11 + B) = 2$. If you dissolve the bracket, you get $5B - B + 10 B - 11 - B = 2$ or $13B - 11 = 2$. If you add 11 on both sides, you get $13B = 13$. If you divide by 13, you get the solution $B = 1$. This information can be inserted into the other equations and solved for the respective missing variable: $10 \times 1 = C$ or $C = 10$, $5 \times 1 = A$ or $A = 5$ and $11 + 1 = D$ or $D = 12$.

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Solution – Exercise 6

$C + D - A = 1$
$5 \times C = D$
$13 - C = A$
$3 \times C - 1 = B$

$A = 11$
$B = 5$
$C = 2$
$D = 10$

The information given in equations two and three for the variables A and D can be inserted into the first equation, so that it can be solved for C: $C + 5C - (13 - C) = 1$. Dissolving the bracket gives $C + 5C - 13 + C = 1$ or $7C - 13 = 1$. Adding 13 on both sides gives $7C = 14$. Dividing by 7 gives the solution $C = 2$. This information can be inserted into the other equations and solved for the respective missing variable: $5 \times 2 = D$ or $D = 10$, $13 - 2 = A$ or $A = 11$ and $3 \times 2 - 1 = B$ or $B = 5$.

Core Module — Latin Squares

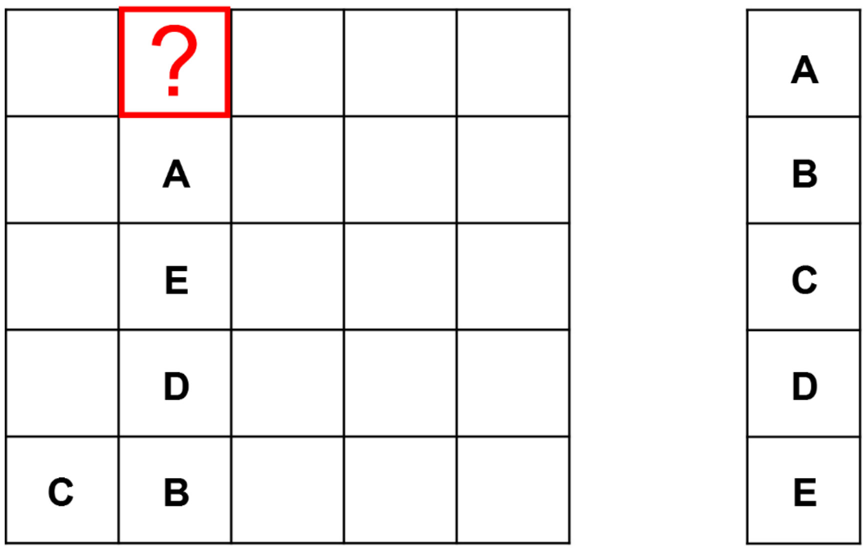
Instructions

In this task you will see a 5x5 grid (a square containing 5 rows and 5 columns).

Some fields of the grid contain letters. Each letter can only appear once in each row and each column. Only the letters that are shown as response options (the row next to the grid) can appear in the grid.

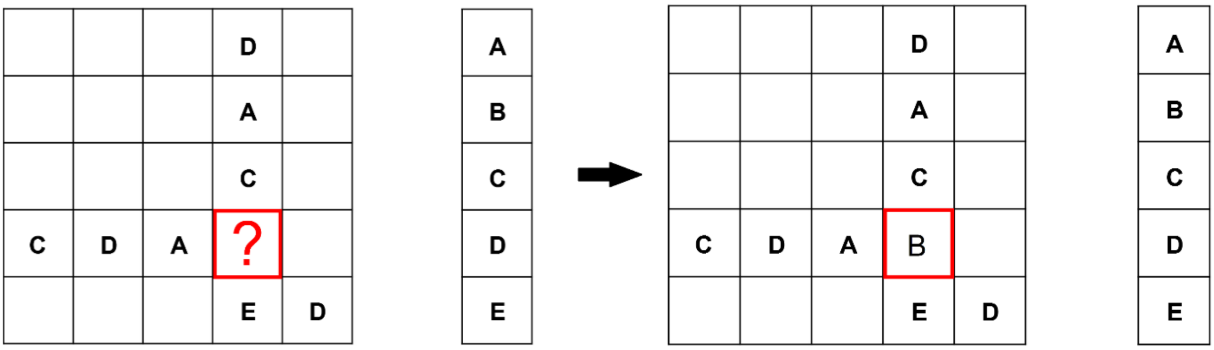
Your task is to decide which letter belongs in the field with the question mark. Sometimes you need to fill in other fields in your mind before you can figure out what letter should replace the question mark.

If you know what the correct solution for the question mark field is, click on the correct response in the solution row.



Next, you will see two examples.

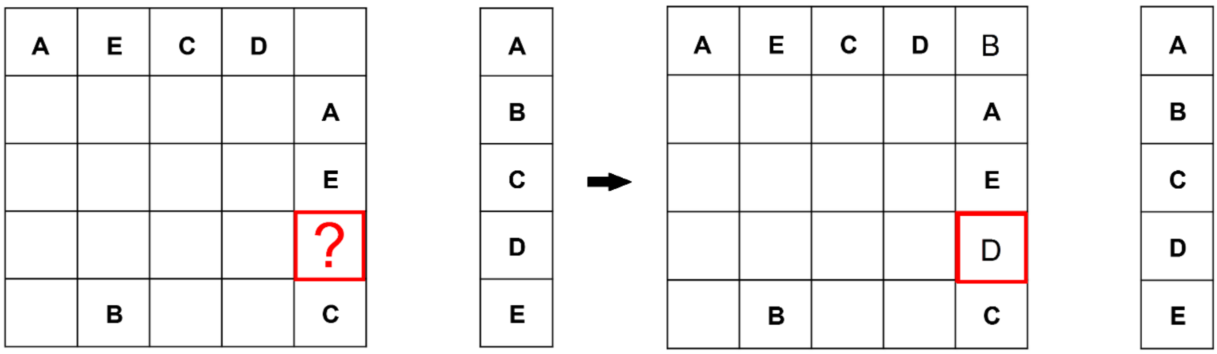
Example 1



Solution of Example 1

In the first example, “B” needs to replace the red question mark, because all other letters D, A, C, and E already appear in this column.

Example 2



Solution of Example 2

In the second example, you first need to fill in “B“ in the first row of the last column. “B” is the only letter, which does not already appear in this row and column. Then you can replace the question mark with “D”, because it is the only letter that does not appear in the last column.

In the exam you have 25 minutes for 20 tasks. Please be as quick and accurate as possible! If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module | Latin Squares

Exercises – Solutions

For the task type **Latin Squares**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Exercise 2 – Difficulty: low

		?		
			D	A
		E		D
A	D			B
D	B		C	

Exercise 3 – Difficulty: medium

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Exercise 4 – Difficulty: medium

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Exercise 5 – Difficulty: high

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Exercise 6 – Difficulty: high

?				C
	D	E	B	A
B		D	A	
	B	C		D

Core Module | Latin Squares
Exercises – Solutions

Note on the solution key

	α	β	γ	δ	ε
1	B	?	A	D	
2	A	B	E	C	
3		A			
4	C				
5	D	E		B	

Solution – Exercise 1

Solution = C

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Solution path:

- In column β, C and D are missing.
- C is already in row 4, so D must be inserted in β4.
- Consequently, C must be inserted in the place of the question mark.

Solution – Exercise 2

Solution = D

		?		
			D	A
		E		D
A	D			B
D	B		C	

Solution path:

- In the place of the question mark, D must be inserted because D is already given in all other columns and rows.

Solution – Exercise 3

Solution = B

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Solution path:

- In column γ, B and C are missing. At position γ4, only B can be inserted, since there is already a B in row 1. This also applies in γ4, or line 4 vice versa for C. Consequently, only a C can be inserted in γ1.
- A and D are missing in column β. A can only be in position β5, because A is already present in row 1. Consequently, only a D can be in position β1.
- From this follows that only one E can be inserted in ε1.

- In row 3 it is now noticeable that A can only be in position ε3, as it is already present in all columns and rows.
- Since a B still has to be inserted in column ε, and there is already a B in line 2, it can only be inserted at the position of the question mark.

Solution – Exercise 4

Solution = D

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Solution path:

- A and D are missing in the first row. A can only be inserted at position α1, since it is already in column γ. Consequently, D must be in position γ1.
- It is now noticeable that D is already present in four different rows and columns and can thus only be used in the place of the question mark.

Solution – Exercise 5

Solution = D

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Solution path:

- Only C can be inserted at position γ4.
- In row 3, A and D are missing. At position γ3, only an A can be inserted because it is already present in column α. Consequently, only a D can be inserted at position α1.
- Only A can be inserted at position β1.
- Only E can be inserted at position α1, as it is already present in all other rows and columns.
- Furthermore, C and B are missing in row 5, whereby only a C is inserted at position α5, since it is already present in column ε. Consequently, there is a B at position ε5.
- In row 1, D and B are still missing. Since B is already in column ε, B must be inserted at position γ1 and D at position ε1.
- At the position of the question mark, D must be inserted, since all other letters are already present in column γ.

Solution – Exercise 6

Solution = E

?				C
	D	E	B	A
B		D	A	
	B	C		D

Solution path:

- At position α3, only a C can be inserted, since all other letters are already in line 3.
- In row 5, A and E are missing. A must be in position α5 because it is already in column δ. Consequently, E is in position δ5.
- In row 4, C and E are missing. Only a C can be inserted at position β4, as it is already present in column ε. Consequently, there is an E at position ε4.
- At position ε2, only a B can be inserted, as all other letters in column ε are already present.
- In column γ, A and B are missing. At position γ1, only a B can be inserted, since there is already a B in the second row. Consequently, A must be inserted in γ2.
- In the first row, A, D and E must be inserted. A must be inserted in β1 because it is already present in all the other columns. Since E is already present in column δ, it must therefore be inserted in the position of the question mark.

Subject Module – Instructions and Exercises

Subject Module

General Instructions

In this task type you see a text and a number of questions which you have to answer. There are 4 answer options for each question.

For each question, there is only one correct solution.

The text, the questions and the answer options may contain figures, tables and formulas.

For working on the entire subject test in the exam, you have 90 minutes in total. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

For practice and illustration of the subject module tasks, three exercises are available here

Redox Reactions

When a substance bonds with oxygen, it is oxidized. An example of *oxidation* is the formation of rust (Fe_2O_3) from the combination of iron (Fe) and oxygen (O_2):

$$4\ Fe + 3\ O_2 \rightarrow 2\ Fe_2O_3$$

When a substance releases oxygen, it is reduced. The *reduction* of mercury oxide is:

$$2\ HgO \rightarrow 2\ Hg + O_2$$

Oxidation and reduction often occur together, e.g. when one reaction partner removes oxygen from another and binds it to itself. When reduction and oxidation reactions occur together, one speaks of *redox reactions*. An example is the reaction of thermite:

$$Fe_2O_3 + 2\ Al \rightarrow 2\ Fe + Al_2O_3$$

The redox concept can also be extended: Reactions in which electrons (e^-) are transferred are also called redox reactions. Here, the *electron donation* reaction is oxidation and the *electron acceptance* reaction is reduction. The formation of common salt serves as an example:

Oxidation: $2\ Na \rightarrow 2\ Na^+ + 2\ e^-$

Reduction: $\underline{Cl_2 + 2\ e^- \rightarrow 2\ Cl^-}$

Redox reaction (the electrons are not shown): $2\ Na + Cl_2 \rightarrow 2\ NaCl$

In general:

Oxidation: $A \rightarrow A^+ + e^-$

Reduction: $\underline{\mathrm{B} + \mathrm{e}^- \rightarrow \mathrm{B}^-}$

Redox reaction: $A + B \rightarrow A^+ + B^-$

Redox reactions occur with complete electron transfers (*ionic bonding*). In addition, the formation of molecules with covalent bonding (*electron pair bonding*) is also a redox reaction. In this case, all components of the molecule are considered individually. Electronegative values are assigned to all bonding electrons so that they can be imagined as charged particles.

The charge of these imaginary ions is referred to as their *oxidation number* and is written above the element symbol. Oxygen is very electronegative and almost always has an oxidation number of -2. Molecules composed of a single element always have an oxidation number of 0. For example, consider water:

$$\overset{0}{2\,H_2} + \overset{0}{O_2} \rightarrow 2\,\overset{+1}{H_2}\overset{-2}{O}$$

$$(4 * 0) + (2 * 0) = 0 = 2 *((2 * 1) + (1 * (-2)))$$

Changes in oxidation numbers are thus an indicator of redox reactions.

Question 1

What happens during the synthesis of water from hydrogen and oxygen?

a) Hydrogen is reduced and oxygen is oxidized.
b) Hydrogen is oxidized and oxygen is reduced.
c) Hydrogen and oxygen are oxidized.
d) Hydrogen and oxygen are reduced.

Question 2

What happens when water is broken down into its elementary components by electrolysis?

a) Hydrogen is reduced and oxygen is oxidized.
b) Hydrogen is oxidized and oxygen is reduced.
c) There is no redox reaction.
d) Hydrogen with the oxidation number -1 and oxygen with the oxidation number +2 are produced.

Question 3

What happens when two single nitrogen atoms (N) react to form a nitrogen molecule (N_2)?

a) One atom is reduced and the other is oxidized.
b) Both atoms are reduced.
c) Both atoms are oxidized.
d) There is no redox reaction.

Question 4

Non-metal oxides react with water to form acids. For example, sulphur dioxide (SO_2) reacts with water to form sulphurous acid (H_2SO_3).

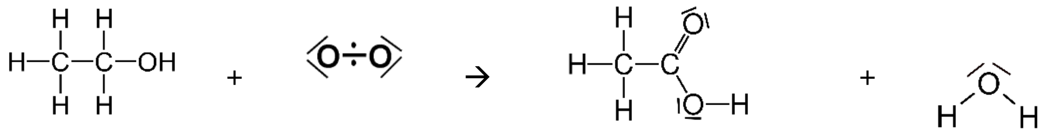
$$\overset{+4\ -2}{SO_2} + \overset{+1\ -2}{H_2O} \rightarrow \overset{+1\ +4\ -2}{H_2SO_3}$$

What is the reaction here?

a) There is only one oxidation.
b) There is only one reduction.
c) There is a redox reaction.
d) There is no redox reaction.

Question 5

During the fermentation of vinegar, ethanol (H_3CCH_2OH) reacts with oxygen (O_2) to form acetic acid (H_3CCOOH) and water:



How can you tell if there is a redox reaction?

a) New molecules are created.
b) The oxidation number of the oxygen atoms of O_2 decreases from 0 to -2.
c) A water molecule is split off from the ethanol.
d) There is no redox reaction because the number of particles does not change.

Question 6

Which formula exclusively describes a reduction reaction?

a) $\mathrm{Cu^{2+} + 2e^- \rightarrow Cu}$
b) $\mathrm{Fe \rightarrow Fe^{2+} + 2e^-}$
c) $\mathrm{2\,H_2 + O_2 \rightarrow 2H_2O}$
d) $\mathrm{2\,Na + Cl_2 \rightarrow 2\,Na^+ + 2\,Cl^-}$

Question 7

Which formula exclusively describes an oxidation reaction?

a) $\mathrm{Cu^{2+} + 2e^- \rightarrow Cu}$
b) $\mathrm{Fe \rightarrow Fe^{2+} + 2e^-}$
c) $\mathrm{O + 2e^- \rightarrow O^{2-}}$
d) $\mathrm{2\,Na + Cl_2 \rightarrow 2\,Na^+ + 2\,Cl^-}$

Subject Modules
Basic Tasks: Solutions

Question 1

Solution: B

During the synthesis of water, hydrogen (H_2) and oxygen (O_2) react to form water (H_2O). In this process, hydrogen loses electrons (oxidation), while oxygen gains electrons (reduction). Since oxidation is the loss of electrons and reduction is the gain of electrons, hydrogen is oxidized, and oxygen is reduced.

Question 2

Solution: A

During electrolysis, water (H_2O) is split into its elemental components, hydrogen (H_2) and oxygen (O_2), using an electric current. At the cathode, hydrogen ions (H^+) gain electrons and form hydrogen gas (reduction). At the anode, oxygen from water molecules loses electrons and forms oxygen gas (oxidation). Since reduction is the gain of electrons and oxidation is the loss of electrons, hydrogen is reduced, and oxygen is oxidized.

Question 3

Solution: D

When two single nitrogen atoms (N) react to form a nitrogen molecule (N_2), they share electrons to form a strong triple covalent bond. Since no electrons are transferred between atoms, there is no oxidation (loss of electrons) or reduction (gain of electrons). Because redox reactions involve electron transfer, and this reaction only involves electron sharing, it is not a redox reaction.

Question 4

Solution: D

In this reaction, sulfur dioxide (SO_2) reacts with water (H_2O) to form sulfurous acid (H_2SO_3). The oxidation states of sulfur (+4), oxygen (-2), and hydrogen (+1) remain the same before and after the reaction. Since a redox reaction involves a change in oxidation states due to electron transfer, and no such change occurs here, this is not a redox reaction but rather an acid formation reaction.

Question 5

Solution: B

In this reaction, ethanol (H_3CCH_2OH) reacts with oxygen (O_2) to form acetic acid (H_3CCOOH) and water. To determine if it is a redox reaction, we analyze the oxidation states of the elements:

1. **Oxygen in O_2 starts at 0** (since it is in its elemental form).
2. **Oxygen in water (H_2O) has an oxidation state of -2**, meaning it has gained electrons (reduction).
3. **The carbon in ethanol is partially oxidized** as it forms acetic acid, meaning it loses electrons (oxidation).

Since oxidation (electron loss) and reduction (electron gain) both occur, this confirms that the reaction is a redox reaction.

Question 6

Solution: A

The given equation **$Cu^{2+} + 2e^- \rightarrow Cu$** represents a reduction reaction because copper ions (Cu^{2+}) gain two electrons ($2e^-$) to form neutral copper (Cu). In a reduction reaction, the oxidation state of an element decreases due to the gain of electrons. Here, copper's oxidation state decreases from **+2 to 0**, confirming that this is exclusively a reduction reaction.

Question 7

Solution: B

The given equation **$Fe \rightarrow Fe^{2+} + 2e^-$** represents an oxidation reaction because iron (Fe) loses two electrons ($2e^-$) to form iron ions (Fe^{2+}). In an oxidation reaction, the oxidation state of an element increases due to the loss of electrons. Here, iron's oxidation state increases from **0 to +2**, confirming that this is exclusively an oxidation reaction.

Electric Circuit

An electric current can be described by three quantities: The current intensity *I*, the voltage *U* and the electrical resistance *R*. The current intensity *I* in the unit Ampere (A) is a measure for the amount of charge that flows through a cross-section in a certain period of time. The electrical voltage *U* in units of volts (V) describes the difference in charge between two points.

When a voltage *U* is applied to an electric circuit, a current of amperage *I* flows in it. Due to interactions in the conductor, an electrical resistance *R* in the unit Ohm (Ω) is created. The resistance in the conductor converts electrical energy into thermal energy.

The relationship between current, voltage and resistance can be described by *Ohm's law*:

$$R = \frac{U}{I}$$

When setting up a circuit, the electrical resistors can be connected in parallel. The total resistance R_{tot} can then be determined by the following formula:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Question 1

A voltage of U = 10 V is applied to a circuit. What is the electrical resistance R when a current intensity of I = 0.2 A is measured?

a) $5\ \Omega$
b) $2\ \Omega$
c) $20\ \Omega$
d) $50\ \Omega$

Question 2

By changing the circuit, the electrical resistance R is doubled. The voltage U = 10 V remains constant. What is the effect of this change?

a) Doubling the electrical resistance generally has no effect.
b) The electric current flows at twice the speed.
c) The current I is half as large.
d) A light bulb connected to the circuit shines brighter.

Question 3

An electrical resistance of R = 5 Ω is present in a circuit. A heating wire is then added and included in the circuit. The heating wire has a resistance of 45 Ω. In order for it to be heated, a current of I = 2.0 A must be reached. What voltage U is required for this?

a) $100\ \mathrm{V}$
b) $25\ \mathrm{V}$
c) $10\ \mathrm{V}$
d) $90\ \mathrm{V}$

Question 4

The same circuit is set up twice. Two different wires are used for this. When an iron wire is used, a lower amperage is measured than when a copper wire is used. Which of the following statements can be correctly deduced from this?

a) When the iron wire is used, the electrical resistance is greater than when the copper wire is used.
b) The electrical conductivity of the copper wire is worse than that of the iron wire.
c) The density of iron is less than the density of copper.
d) There is a linear relationship between the conductivity of the two metals.

Question 5

When setting up a circuit, the electrical resistors can be connected in parallel. The total resistance R_{tot} can then be determined by the following formula:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

R_1, R_2 and R_3 are the values of the individual partial resistances. So which of the following statements is correct?

a) Each additional parallel resistor worsens the conductivity of the circuit.
b) An increase of the applied voltage leads to a lower current.
c) The total resistance $\mathrm{R_{tot}}$ is smaller than each of the individual partial resistances $(R_1, R_2, R_3, \ldots)$.
d) The order of the partial resistors has a great influence on the total resistance.

Question 6

Two resistors $R_1 = 4\,\Omega$ and $R_2 = 6\,\Omega$ are connected in parallel. What is the total resistance R_{tot} resulting from this?

a) $10\,\Omega$
b) $5\,\Omega$
c) $12\,\Omega$
d) $2{,}4\,\Omega$

Question 7

Two resistors $R_1 = 4\ \Omega$ and $R_2 = 12\ \Omega$ are connected in parallel. What is the total resistance R_{tot} resulting from this?

a) $16\ \Omega$
b) $8\ \Omega$
c) $3\ \Omega$
d) $10\ \Omega$

Question 1

Solution: D

The solution is correct because the relationship between voltage (U), current (I), and resistance (R) is given by Ohm's law:

$$R = \frac{U}{I}$$

Substituting the values:

$$R = \frac{10\,V}{0.2\,A} = 50\,\Omega$$

Thus, the electrical resistance is 50 ohms.

Question 2

Solution: C

According to Ohm's law if the resistance R is doubled while the voltage U remains constant, the current I must decrease to maintain the equation. Specifically, if R doubles, then I is halved. This is because:

$$I = \frac{U}{R}$$

So, with the resistance doubled, the current is reduced by half.

Question 3

Solution: A

To find the required voltage, we use Ohm's law:

$$U = I \times R$$

In this case, the total resistance in the circuit is the sum of the individual resistances:

$$R_{total} = R_{resistance} + R_{heating\ wire} = 5\ \Omega + 45\ \Omega = 50\ \Omega$$

Now, applying Ohm’s law:

$$U = I \times R_{total} = 2.0\ \mathrm{A} \times 50\ \Omega = 100\ \mathrm{V}$$

So, a voltage of 100 V is required for the current to reach 2.0 A.

Question 4

Solution: A

According to Ohm's law, the current I is inversely proportional to the resistance R for a given voltage U:

$$I = \frac{U}{R}$$

If the amperage (current) is lower when the iron wire is used compared to the copper wire, it means the resistance of the iron wire must be higher. This is because, for the same applied voltage, a higher resistance results in a lower current. Therefore, the iron wire has a greater electrical resistance than the copper wire.

Question 5

Solution: C

When resistors are connected in parallel, the total resistance R_{tot} is always smaller than the resistance of any individual resistor in the circuit. This is due to the formula for parallel resistances:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

Since the reciprocals of the individual resistances are added, the total resistance is always less than the smallest of the individual resistances. This leads to a total resistance that is smaller than any of the partial resistances $R_1, R_2, R_3, \dots$.

Question 6

Solution: D

The total resistance for resistors connected in parallel is given by the formula:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Substituting the values $R_1 = 4\,\Omega$ and $R_2 = 6\,\Omega$:

$$\frac{1}{R_{tot}} = \frac{1}{4} + \frac{1}{6}$$

To simplify:

$$\frac{1}{R_{tot}} = \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$$

Now, invert the result to find R_{tot}:

$$R_{tot} = \frac{12}{5} = 2.4\,\Omega$$

Thus, the total resistance is 2.4 ohms.

Question 7

Solution: C

The total resistance for resistors connected in parallel is calculated using the formula:

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Substituting the values $R_1 = 4\ \Omega$ and $R_2 = 12\ \Omega$:

$$\frac{1}{R_{tot}} = \frac{1}{4} + \frac{1}{12}$$

Finding a common denominator (12):

$$\frac{1}{R_{tot}} = \frac{3}{12} + \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$$

Now, inverting the result:

$$R_{tot} = 3\ \Omega$$

Thus, the total resistance is 3 ohms.

Datatypes

In programming languages, *variables* are names given to memory locations in which you can store values. Most programming languages use typed variables, i.e. each variable has one of several possible data types. In the Java language, for example, the following data types exist (extract):

- **boolean** – Values **false** und **true**, memory requirement of 1 bit
- **short** – integer numbers from -2^{15} bis $2^{15}-1$, memory requirement of 16 bit
- **int** – integer numbers from -2^{31} bis $2^{31}-1$, memory requirement of 32 bit
- **float** – fractional so-called floating-point numbers up to maximum of approximately $\pm 10^{38}$, memory requirement of 32 bit
- **double** – floating-point numbers up to max. approx. $\pm 10^{308}$, memory requirement of 64 bit
- **string** – character string, sequence of characters of any length, memory requirement depends on length

In a Java program, a variable can be defined by indicating the type and the name of the variable. A location in memory is then reserved. If a value is also specified, this memory is initialized with this value, e.g.:

```
int x = 5;
string s = "Hello ☺";
```

These two examples show how integer constants and strings are written. Floating-point numbers consist of digits with a decimal point and/or an exponent, which has a similar meaning as the exponent representation on the calculator:

```
float f1 = 5.0;
float f2 = +5e0;
float f3 = +50e-1;
float f4 = 5.00000e0;
float f5 = 5f;
```

Alternatively, a small "f" can be appended to a number to indicate the type **float** (small d for type **double**). So all of these five variables have the same floating-point type value of 5.

Calculations are performed in Java only in **int** and **double**. This means that there is an implicit type conversion to one of these types, e.g., **short** to **int** and **float** to **double**, when any calculation is performed. If two operands are of different types, then their types are unified, i.e., **int** becomes **double**. An implicit type conversion also takes place during an assignment.

```
float f6 = 5;
float f7 = 2 * 2.5;
```

At **f6**, the type **int** is implicitly converted to the type **float**. In **f7** we have a multiplication of an **int** and a **double** value. The **int** value is first converted to **double** and then multiplied. The result is then implicitly converted to **float**.

Calculations in type **int** usually return an int result, i.e., the division is also only integer and ignores the fractional part.

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Question 1

You have a variable "married" that should store whether a person is married, a variable "age" that gives the age of a person, and a variable "name" that contains the name of the person. Which of the following types are most appropriate?

a) **boolean married**; **float age**; **string name**;
b) **int married**; **float age**; **string name**;
c) **boolean married**; **short age**; **string name**;
d) **short married**; **short age**; **double name**;

Question 2

Two values from two **float** variables are added. The calculation is done in which type?

a) **boolean**
b) **float**
c) **int**
d) **double**

Question 3

Two values from two short variables are divided. The calculation is done in which type?

a) **float**
b) **int**
c) **double**
d) **short**

Question 4

The following Java program is given:

```
short s = 4;
float x = 3 + s/3;
```

What is the value of the variable **x** after its assignment?

a) 4,33333333333sd
b) 4
c) 3
d) 4,25

Question 5

You need 1000 **float** variables. How much memory do they require?

a) 16000 Bit = 2000 Byte
b) 1000 Bit = 125 Byte
c) 32000 Bit = 4000 Byte
d) 32768 Bit = 4096 Byte

Question 6

The following Java program is given:

```
int i = 2;
double d = (-i)*(1/i)+1f;
```

What is the value of the variable **d** after its assignment?

a) -1
b) 0
c) 2
d) 1

Question 1

Solution: C

For the variable "married" it is enough to store *true* or *false*. For the "name", the file type *string* is best. The "age" is usually stored in integer numbers. The file type *short* is therefore the most efficient way to store the age.

Question 2

Solution: D

In Java, calculations are performed only in *int* or *double*. Since two *float* variables are added, they are converted to *double*.

Question 3

Solution: B

In Java, calculations are performed only in *int* or *double*. Since two *short* variables are divided, they are converted to *int*.

Question 4

Solution: B

The calculation with the *short* variable s = 3 is implicitly converted to *int*. Only integer numbers can be stored via *int*. Therefore, a 1 is stored for s/3. Adding to x = 3 results in 4.

Question 5

Solution: C

The memory requirement of a *float* variable is 32 bits. 1000 variables therefore require 32000 bits. 8 bits are 1 byte.

Question 6

Solution: D

The expression in the second parenthesis stands for a fraction or a decimal number. However, since only integer numbers can be stored in an *int* variable, only the first part of the number is stored, i.e. 0. The product therefore also becomes 0. If a 1 (1f) is then added, the result is 1.

The Fourier Series

Periodic signals are basic patterns that repeat over time.

The period *p* is the time a signal needs to complete a cycle:

$$f(t+p) = f(t)$$

Many phenomena examined by engineers and scientists are periodical (for example: electricity and electric tension in alternating current circuits). Trigonometric functions are used to model these signals. For this purpose, several trigonometric functions are added to one another. The sum of these trigonometric functions is called the Fourier series. The bigger *n* is, the more the Fourier series is adapted to the modeled signal:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi t}{L}) + \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi t}{L})$$

Period	$p = 2L$
Constant	$\pi \approx 3{,}14$
$\frac{a_0}{2}$	mean value of the model.

a_n and b_n are Fourier coefficient 1 and 2. They adapt the model to the signal.

Some Fourier series can be simplified. For this purpose, one needs to know if the function is even or odd. For an even function, the coefficients b_n will be zero. For odd functions, the coefficients a_0 and a_n will be zero.

The graph of an even function is always symmetric to the y-axis (see Figure 1):

$$f(-t) = f(t)$$

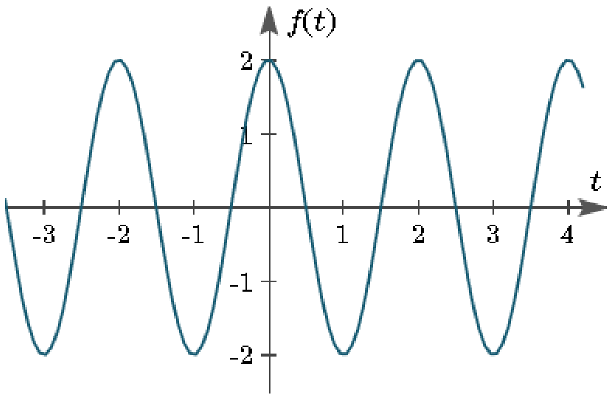


Figure 1. Example of an even function ($a_0 = 0,\ a_1 = 2,\ a_{2,3,\dots} = 0,\ b_{1,2,\dots} = 0$)

The graph of an odd function is always symmetric to its origin.

$$f(-t) = -f(t)$$

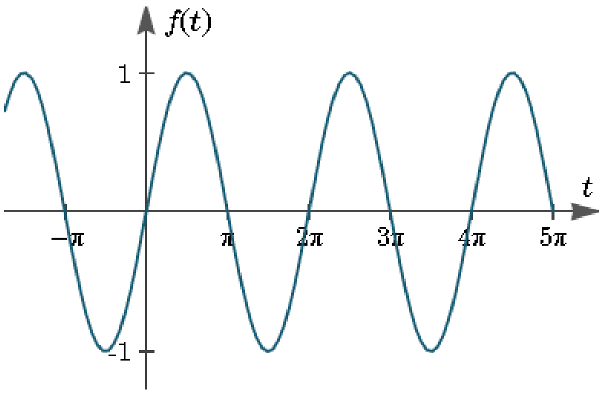
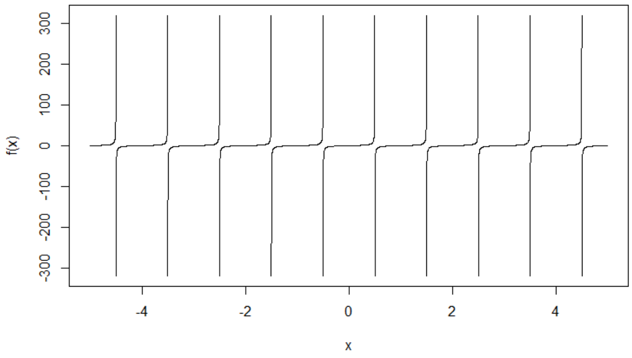


Figure 2. Example of an odd function ($a_0 = 0,\ a_{1,2,\dots} = 0,\ b_1 = 1, b_{2,3,\dots} = 0$)

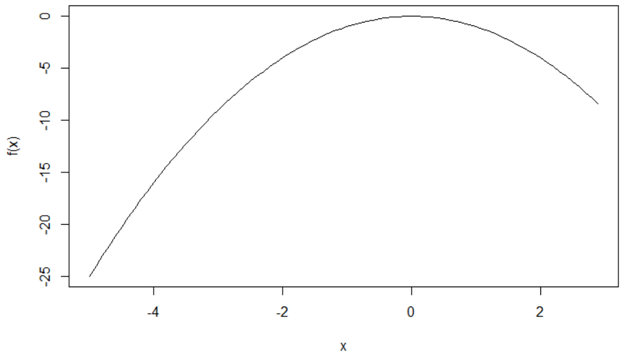
Question 1

Which graph shows a periodic signal?

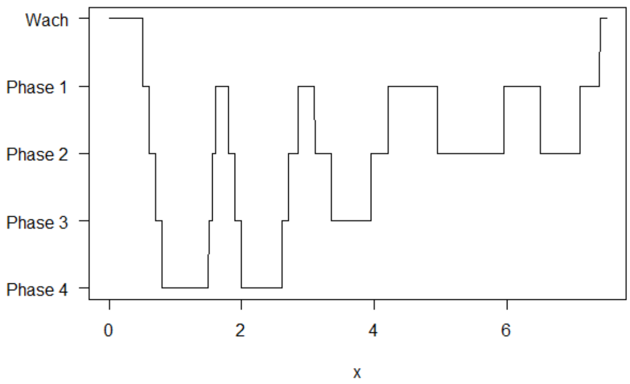
Graph 1



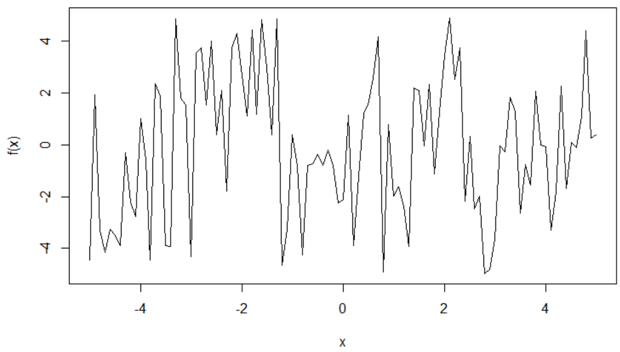
Graph 2



Graph 3



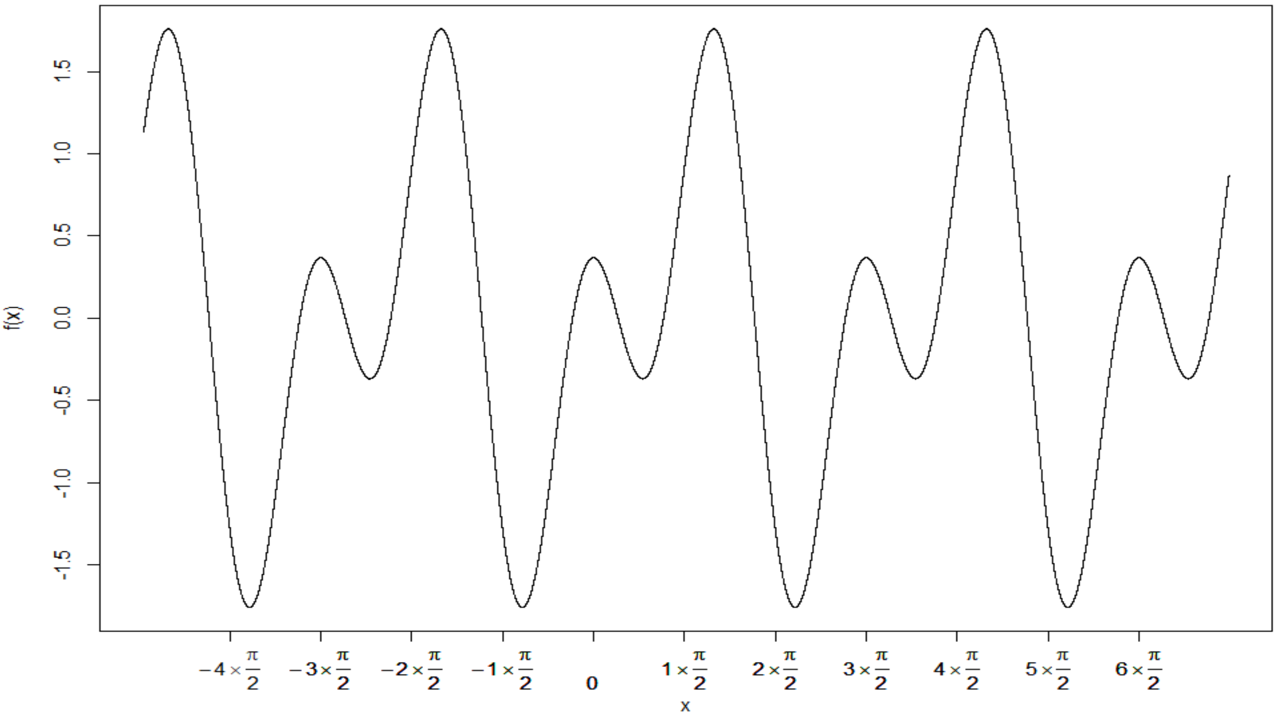
Graph 4



a) Graph 1
b) Graph 2
c) Graph 3
d) Graph 4

Question 2

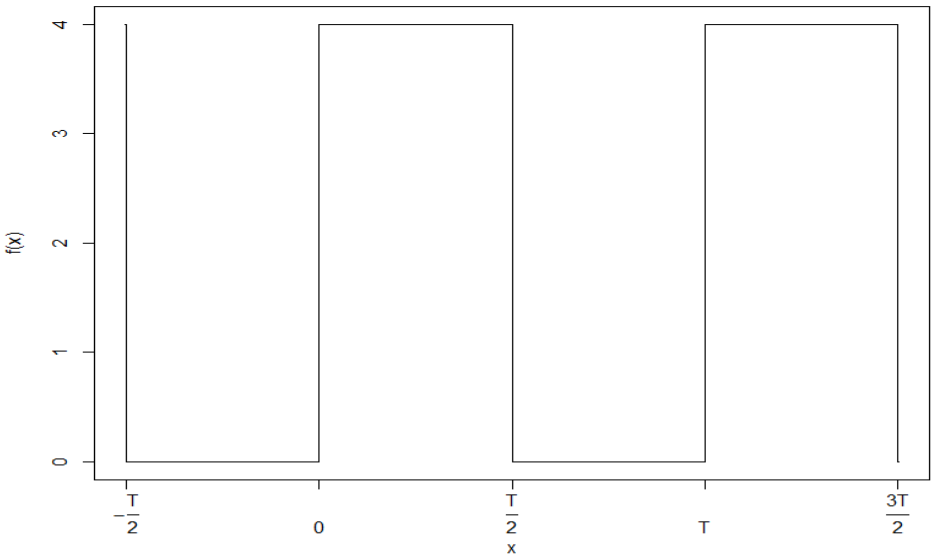
What is the period *p* of the signal below?



a) π
b) $\frac{\pi}{2}$
c) $3\frac{\pi}{2}$
d) $4\frac{\pi}{2}$

Question 3

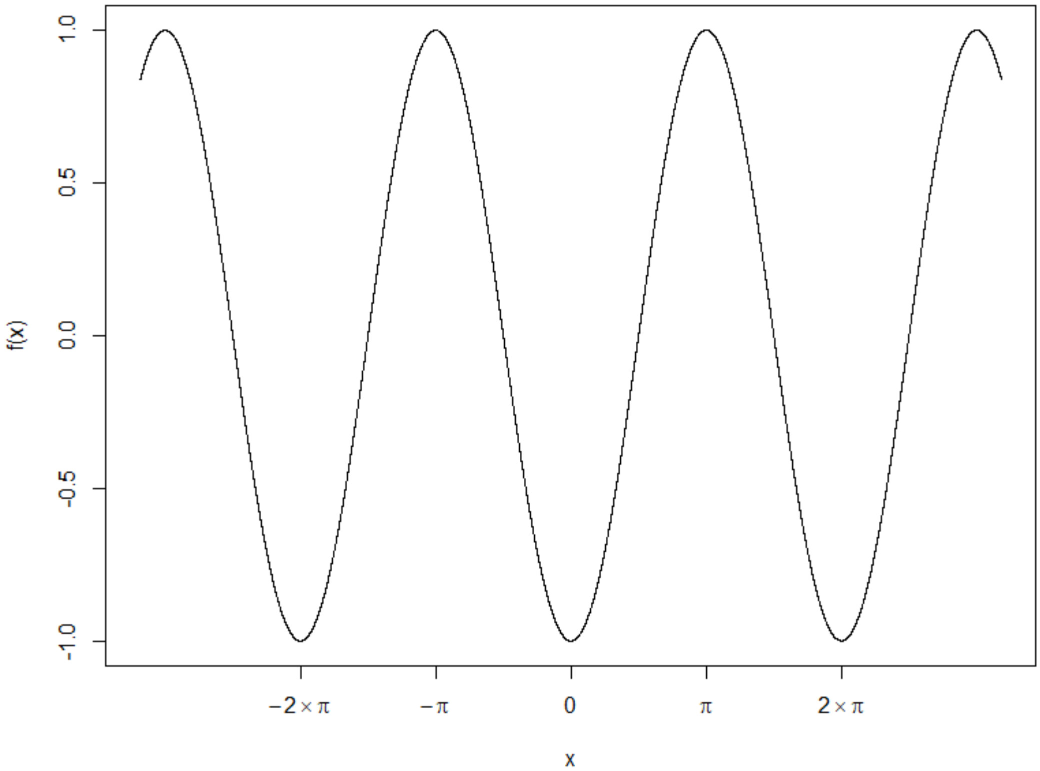
Which value does a_0 have when this function is modeled by a Fourier series?



a) a_0 = 0
b) a_0 = 2
c) a_0 = 4 / π
d) a_0 = T / 2

Question 4

Which statement about function f(x) is correct?



a) f(x) has p = 2π
b) f(x) is an odd function
c) f(x) is not a periodic function
d) f(x) has a mean value of π

Question 5

Which of the following functions can represent a Fourier series of an odd function?

Function 1: $f(x) = \frac{1}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\cos\left(\frac{n\pi x}{L}\right) + -\frac{1}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right)$

Function 2: $f(x) = \frac{4}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right)$

Function 3: $f(x) = \frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right)$

Function 4: $f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1,3,5\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right)$

a) Function 1
b) Function 2
c) Function 3
d) Function 4

Question 1

Solution: A

Graph A shows a periodic function, as it exhibits a repeating pattern over regular intervals along the x-axis. The function has vertical asymptotes, indicating a periodic nature with discontinuities at specific points.

A periodic function is one that satisfies $f(x+T)=f(x)$ for some period T. In the given graph, the function repeats its behavior consistently, confirming its periodicity.

Question 2

Solution: C

The period p of a periodic function is the smallest positive value for which the function repeats itself, meaning $f(x+p)=f(x)$.

From the given graph, we observe that the pattern of peaks and troughs repeats at intervals of $3\frac{\pi}{2}$. This suggests that the function has a fundamental period of $3\frac{\pi}{2}$, as the shape of the waveform remains consistent over this interval. Thus, the correct answer is:

$$p=\frac{3\pi}{2}$$

Question 3

Solution: B

The value a_0 in a Fourier series is just the average (mean) of the function over one full period. In this case, the function is **4 for half the time** and **0 for the other half**. The average value is:

$$\frac{4+0}{2}=2$$

So, $a_0=2$.

Question 4

Solution: A

A function is periodic if it repeats itself after a certain interval, called the period p.

From the graph, we see that the function completes one full cycle from $\mathbf{0}$ **to** $\mathbf{2\pi}$ and then repeats. This means that shifting the function by 2π does not change it, confirming that the period is:

$$p = 2\pi$$

Thus, the correct statement is:
$\boldsymbol{f(x)}$ has $\boldsymbol{p = 2\pi}$

Question 5

Solution: B

A Fourier series of an **odd function** consists only of **sine terms**, because sine functions are themselves odd $(sin(-x) = -sin(x))$. Odd functions have **no cosine terms** in their Fourier series.

Function 2 contains **only sine terms**, making it a valid Fourier series representation of an odd function.

Synthetic Materials

Synthetic materials are divided into thermoplastics, thermosets and elastomers according to their physical properties.

Depending on their composition, **thermoplastics** can already be deformed at room temperature. At temperatures between 80 and 160 degrees, they become soft and can be completely reshaped by various processes. Thermoplastics retain their new shape after they have cooled down. When heated again they become so soft, that their new shape is lost again.

Thermosets are hardened synthetic materials. They remain hard even at higher temperatures. They do not burn or melt and cannot be welded. Above a certain temperature range, they decompose. Their shape can only be changed by machining (comparable to woodworking).

Elastomers can change their shape for a short time by pressure or elongation. After the application of force, they return to their original shape. They are largely insoluble in solvents and do not soften when heated.

In the synthetic production of plastics from so-called *monomers* (reactive molecules), a distinction is made between three types:

Polymerisation*:* Monomers react by cleavage and reattachment to form bonds of polymers. Pressure and heat turn the multiple bonds of the monomers into single bonds. The single bonds allow the monomers to combine to form polymers (*chain polymerization).*

Polycondensation*:* Monomers react to form polymers by splitting off simple molecules (e.g. water).

Polyaddition*:* Monomers are linked together additively (without splitting off certain molecules as in polycondensation).

Question 1

How can you check whether the material is thermoset or thermoplastic?

a) By application of force (e.g. with a hammer)
b) With solvents
c) By heating
d) By cooling

Question 2

What process is polymerisation?

a) During the exchange of protons, individual molecules combine to form a giant molecule.
b) When the same molecules merge, water is split off.
c) Monomers react by splitting complex molecules into multiple bonds.
d) By adding energy, connections are split and new ones are created.

Question 3

Which everyday object is a thermoplastic?

a) Pan
b) Yoghurt cups
c) Reflectors in car headlights
d) Fire helmet

Question 4

Which everyday object is a thermoset?

a) Ballpoint pen
b) Shower curtain
c) Vinyl record
d) Power socket

Question 5

Which one of the following statements on the characteristics of synthetic materials is correct?

a) Due to their molecular structure, thermosets can be processed with tools.
b) Thermosets can change their shape by heating and cooling.
c) Thermoplastics are brittle and crack when subjected to force.
d) Thermoplastics do not deform as easily under heat as thermosets.

Subject Modules

Basic Tasks: Solutions

Question 1

Solution: C

The best way to distinguish between a thermoset and a thermoplastic material is by heating it. Thermoplastics melt, while thermosets char or burn due to their cross-linked structure.

Question 2

Solution: D

Polymerisation is the process where small molecules (monomers) link together to form long chains (polymers). This happens when energy is added, breaking some existing bonds and allowing new stronger connections to form between monomers.

Question 3

Solution: B

Yoghurt cups are made from thermoplastic materials like polypropylene (PP) or polystyrene (PS), which soften when heated and can be reshaped. This property makes them easy to manufacture and recycle.

Question 4

Solution: D

Power sockets are made from thermoset materials like bakelite or melamine, which do not melt when heated. Their strong cross-linked structure makes them heat-resistant and durable, ensuring safety in electrical applications.

Question 5

Solution: A

Thermosets have a strong cross-linked molecular structure, making them hard and durable. This allows them to be shaped and processed with tools, but unlike thermoplastics, they cannot be melted and reshaped after curing.

Acid-Base Reactions

The *autoprotolysis* of water refers to the self-ionization of water molecules, where two water molecules react to form hydronium ions (H_3O^+) and hydroxide ions (OH^-). This reaction can be written as:

$$2\,H_2O\;_{(l)} \rightleftharpoons H_3O^+{}_{(aq)} + OH^-{}_{(aq)}$$

The *law of mass action* can be applied to this equilibrium reaction to describe the relationship between the concentrations of the ions and the equilibrium constant:

$$K = \frac{[H_3O^+]\,[OH^-]}{[H_2O]^2}$$

At 25 °C the concentration of water is around $55\,\frac{mol}{L}$, whereas the concentration of $[H_3O^+]$ and $[OH^-]$ can be measured to around $10^{-7}\,\frac{mol}{L}$. Therefore, the change in the water concentration can be neglected. It can be seen as constant and multiplied by K to obtain the ionization constant (or ion product of water, K_w), which is:

$$K_w = K \cdot [H_2O] = [H_3O^+]\,[OH^-]$$

The same scheme can be followed for a base-acid reaction in aqueous solution:

$$HA_{(aq)}\;H_2O\;_{(l)} \rightleftharpoons A^-{}_{(aq)} + H_3O^+{}_{(aq)}$$

With the acid constant

$$K_a = K \cdot [H_2O] = \frac{[H_3O^+][A^-]}{[HA]}$$

The pH is defined as the negative decadic logarithm of the H_3O^+- ion concentration, which can be obtained by transforming the equation to the so-called *Henderson-Hasselbalch equation*:

$$pH = pK_a + \log\frac{[A^-]}{[HA]} = pK_a - \log\frac{[HA]}{[A^-]}$$

For strong acids, such as hydrochloric acid or nitric acid, the pH can simplified be expressed as

$$pH = -\log[H_3O^+] = -\log[HA].$$

Question 1

What is the ion product of water at 25 °C?

a) $10^{-7}\frac{\mathrm{mol}}{\mathrm{L}}$

b) $10^{-7}\frac{\mathrm{mol}^2}{\mathrm{L}^2}$

c) $10^{-14}\frac{\mathrm{mol}}{\mathrm{L}}$

d) $10^{-14}\frac{\mathrm{mol}^2}{\mathrm{L}^2}$

Question 2

How can the concentration of the hydroxide ions be calculated?

a) $[\mathrm{OH^-}] = \sqrt{\mathrm{K}}$

b) $[\mathrm{OH^-}] = \sqrt{\mathrm{K_W}}$

c) $[\mathrm{OH^-}] = \frac{1}{2}\,[\mathrm{H_2O}]$

d) $[\mathrm{OH^-}] = \sqrt{[\mathrm{H_2O}]}$

Question 3

Indicators (Ind) are weak organic acids or bases, which, in solution, change their colour according to the pH of the solution:

$$\mathrm{HInd} + \mathrm{H_2O} \rightleftharpoons \mathrm{H_3O^+} + \mathrm{Ind^-}$$

How can the pH of an indicator that is solved in water be calculated?

a) $\mathrm{pH} = \mathrm{pK_{Ind}} - \log\frac{[\mathrm{HInd}]}{[\mathrm{Ind^-}]}$

b) $\mathrm{pH} = \mathrm{pK_{Ind}} - \log\frac{[\mathrm{Ind^-}]}{[\mathrm{HInd}]}$

c) $\mathrm{pH} = \mathrm{pK_{Ind}} - \log\frac{[\mathrm{H_3O^+}]}{[\mathrm{H_2O}]}$

d) $\mathrm{pH} = \mathrm{pK_{Ind}} - \log\frac{[\mathrm{H_3O^+}]}{[\mathrm{H_2O}]}$

Question 4

Buffer solutions contain a mixture of

- a weak acid and its conjugate (corresponding) base (or the respective salt)
- or a weak base and its conjugate (corresponding) acid.

The factor determining the pH value is the ratio or protolysis equilibrium of the buffer pair. According to the Henderson-Hasselbalch equation, which statement is correct?

a) When acid and conjugate base are present in the same concentration, the pH equals the pK_a value of the acid.
b) When the concentration of the acid is a decimal of the conjugate base, the pH equals the pK_a value of the acid.
c) When decreasing the concentration of the conjugate base, it is possible to add more acid while maintaining a buffering effect of the pH.
d) When increasing the concentration of the acid, it is possible to add more acid while maintaining a buffering effect of the pH.

Question 5

A pH curve is recorded for the titration of 20 mL HCl with 0.1 M NaOH. The equivalent point is at V(NaOH) = 10 mL. What was the concentration of the HCl stock solution?

a) c(HCl) = 0.02 mol/L
b) c(HCl) = 0.002 mol/L
c) c(HCl) = 0.05 mol/L
d) c(HCl) = 0.005 mol/L

Subject Module	Chemistry
Advanced Task 1: Solutions	

Question 1

Solution: D

Insert the given concentration of H_3O^+ and OH^-in the formula of the ion product of water:

$$K_w = [H_3O^+]\,[OH^-] = 10^{-7}\frac{mol}{L} \cdot 10^{-7}\frac{mol}{L} = \mathbf{10^{-14}\frac{mol^2}{L^2}}$$

Question 2

Solution: B

From the autoprotolysis reaction of water

$$2\,H_2O_{\,(l)} \rightleftharpoons H_3O^+{}_{(aq)} + OH^-{}_{(aq)}$$

One can deduce that as one hydroxide ion is formed, a hydronium ion is formed at the same time. Therefor the concentration of H_3O^+ and OH^- is the same and the ion product can also be written as

$$K_w = [OH^-]^2$$

Solving that for $[OH^-]$ one obtains

$$K_w = [OH^-]^2 \Leftrightarrow \mathbf{[OH^-]} = \sqrt{\mathbf{K_W}}$$

Question 3

Solution: A

As Indicators are weak acids or bases, the Henderson-Hasselbalch equation is needed to calculate the pH of the solution. Insert the concentration of the indicator as the concentration of the acid and respectively the concentration of the conjugate base:

$$\mathbf{pH} = pK_a - \log\frac{[HA]}{[A^-]} = \mathbf{pK_{Ind} - \log\frac{[HInd]}{[Ind^-]}}$$

Question 4

Solution: A

Inserting the same concentration of acid and conjugate base in the Henderson-Hasselbalch equation results in a **pH value which is equal to the pK$_a$**.

$$\mathbf{pH} = \mathrm{pK_a} - \log\frac{[\mathrm{HA}]}{[\mathrm{A^-}]} = \mathrm{pK_a} - \log 1 = \mathrm{pK_a} - 0 = \mathbf{pK_a}$$

Question 5

Solution: C

HCl is considered to be a strong acid and NaOH a strong base. It can be assumed that their corresponding ions do not influence the pH. Therefore, the equivalent point is also the neutral point and one can conclude that there are as many HCl molecules as NaOH molecules:

$$\mathrm{n(HCl)} = \mathrm{n(NaOH)} = \mathrm{c(NaOH)} \cdot \mathrm{V(NaOH)} = 0.1\,\frac{\mathrm{mol}}{\mathrm{L}} \cdot 0.01\ \mathrm{L} = 0{,}001\ \mathrm{mol}$$

The concentration can be calculated with the volume of the stock solution:

$$\mathrm{c(HCl)} = \frac{\mathrm{n(HCl)}}{\mathrm{V(HCl)}} = \frac{0.001\ \mathrm{mol}}{0.02\ \mathrm{L}} = 0.05\,\frac{\mathrm{mol}}{\mathrm{L}}$$

Infrared Spectroscopy

Infrared spectroscopy (*IR spectroscopy*) is a powerful analytical technique used to identify and study chemicals based on the absorption of infrared light. The principle behind IR spectroscopy is that molecules absorb specific frequencies of infrared light, causing vibrations of the bonds within the molecules. These vibrations are directly related to the structure and functional groups of the molecule. If the molecule undergoes a change in dipole moment during a vibrational motion, it is IR active.

In the molecule, irradiation with infrared light leads to the excitation of specific vibrational modes. The process is governed by quantum mechanics, particularly the concept of vibrational energy levels and selection rules. In this context, the molecule absorbs IR light only at specific frequencies that match the natural frequency of vibration of the bonds within the molecule. These fundamental vibrational frequencies ν_0 are influenced by the mass of the atoms involved in the bond (in form of the reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$) and the strength (force constant k) of the bond.

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

The frequency ν_0 corresponds to the energy difference between the vibrational energy levels, which can be described by the quantum harmonic oscillator model:

$$E_{vib} = \left(\nu + \frac{1}{2}\right) h\nu_0$$

h is Planck's constant and

$\nu = 0, 1, 2, \ldots$ is the vibrational quantum number. The allowed transitions for a harmonic oscillator occur when $\Delta\nu = \pm 1$.

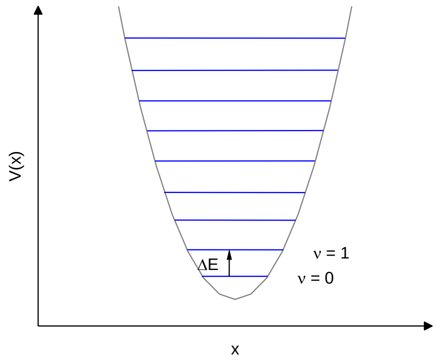


Figure 1. Potential curve of the harmonic oscillator. Each vibrational mode can be associated to an energy level.

For linear molecules, the total transition energy includes contributions from both vibrational and rotational changes:

$$\Delta E = \Delta E_{vib} + \Delta E_{rot}$$

The rotational energy levels are described by

$$E_{rot} = B \cdot J(J+1)$$

Where B is the rotational constant and $J = 0, 1, 2, \ldots$ is the rotational number. Possible rotational transitions $\Delta J = \pm 1$ superimpose vibrational transitions and create a characteristic splitting of the IR spectrum in two branches:

- The P-branch originating from transitions $\Delta J = -1$, which can be observed at frequencies lower than ν_0.
- The R-branch originating from transitions $\Delta J = +1$, which can be observed at frequencies higher than ν_0.

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Question 1

Which of the following samples would not show up in an IR spectrum?

a) A sample with a high degree of symmetry (e. g. CCl_4)

b) A sample with only or mainly hydrogen and carbon atoms (e. g. propane)

c) A sample containing functional groups (e. g. $-NH_2$ or $-SO_2$)

d) A sample with distinct polar bonds (e. g. H_2O or CH_3OH)

Question 2

On which two key factors does the IR absorption frequency for a diatomic molecule depend?

a) Dipole moment of the molecule and electron density

b) Bond strength and atomic masses of the two atoms in the bond

c) Dipole moment of the molecule and atomic masses of the two atoms in the bond

d) Bond strength and electron density

Question 3

What aspects of vibration are considered, when modelling vibration using the rigid rotator and harmonic oscillator models?

a) The change in bond length during vibration

b) The non-periodic oscillation of atoms

c) The coupling between rotational and vibrational modes

d) The quantization of energy level

Question 4

The dependence of the characteristic frequency ν_0 on the mass µ and the force constant k (which increases with greater bond strength) can be used to identify molecular bonds and functional groups of molecules. Put the following chemical bonds in the correct order. Sort them from low frequency ν_0 to high ν_0.

a) $\nu_0(\mathrm{C} \equiv \mathrm{C}) < \nu_0\,(\mathrm{C} = \mathrm{C}) < \nu_0\,(\mathrm{C} = \mathrm{O}) < \nu_0\,(\mathrm{S} = \mathrm{O})$

b) $\nu_0(S = O) < \nu_0\,(\mathrm{C} = \mathrm{O}) < \nu_0\,(\mathrm{C} = \mathrm{C}) < \nu_0\,(\mathrm{C} \equiv \mathrm{C})$

c) $\nu_0(C = C) < \nu_0\,(\mathrm{C} = \mathrm{O}) < \nu_0\,(\mathrm{S} = \mathrm{O}) < \nu_0\,(\mathrm{C} \equiv \mathrm{C})$

d) $\nu_0(\mathrm{C} = \mathrm{O}) < \nu_0\,(\mathrm{C} \equiv \mathrm{C}) < \nu_0\,(\mathrm{C} = \mathrm{C}) < \nu_0\,(\mathrm{S} = \mathrm{O})$

Deuterium (D) is the natural hydrogen isotope $^2_1\mathrm{H}$.

Question 5

A carbon monoxide molecule undergoes a vibrational transition with the rotational quantum number changing from J=1 to J=0. What is observed in the IR spectrum and why?

a) A peak in the P-branch at a frequency lower than ν_0, because of a loss of rotational energy.

b) A peak in the P-branch at a frequency lower than ν_0, because of an increase in rotational energy

c) A peak in the R-branch at a frequency higher than ν_0, because of a loss of rotational energy

d) A peak in the R-branch at a frequency higher than ν_0, because of an increase in rotational energy

Question 1

Solution: A

The requirement for a sample to generate an IR-signal is that it undergoes a change in the dipole moment during the vibration. For **molecules with a high degree of symmetry, any vibrational motion does not produce a net change in the dipole moment.**

Question 2

Solution: B

The IR absorption frequency ν_0 is given by

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mu}}$$

And therefore influenced by the (reduced) **masses** μ and the force constant k, which represents the **bond strength** of the atoms.

Question 3

Solution: D

The rigid rotator model assumes a fixed bond length and angle, implying that the molecule does not undergo any deformation during rotation. In reality, molecules can distort during vibration, and bond lengths and angles change, which is not accounted for in the rigid rotator approximation. Similarly, the harmonic oscillator model assumes that atoms oscillate around equilibrium positions without considering the effects of bond stretching, bending, or interactions with other molecules.

Furthermore, the anharmonicity of vibration is neglected, as the harmonic oscillator model assumes that the potential energy of a vibrating system is purely quadratic.

Both the rigid rotator and harmonic oscillator models often treat modes as independent, but in real molecules, vibrational modes can couple with one another.

In the rigid rotator model, **quantization applies to the rotational energy levels.** The energy associated with the rotation of a molecule is quantized, meaning that only certain discrete rotational energy levels are allowed. The harmonic oscillator model considers the **quantization of energy levels for vibrational modes**.

Question 4

Solution: B

Looking again at the formula for the vibrational frequency

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{\mathrm{k}}{\mu}}$$

One can deduce that the frequency increases with stronger atomic bonds and lower atomic masses. This means the sulphur-oxygen double bond typically has a lower vibrational frequency than the carbon-oxygen double bond because sulphur is heavier than carbon. The carbon-carbon triple bond is stronger and stiffer than the carbon-carbon double bond, so the triple bond will have a higher vibrational frequency and the correct order is

$$\boldsymbol{\nu_0(S = O) < \nu_0\,(\mathrm{C} = \mathrm{O}) < \nu_0\,(\mathrm{C} = \mathrm{C}) < \nu_0\,(\mathrm{C} \equiv \mathrm{C})}$$

Question 5

Solution: A

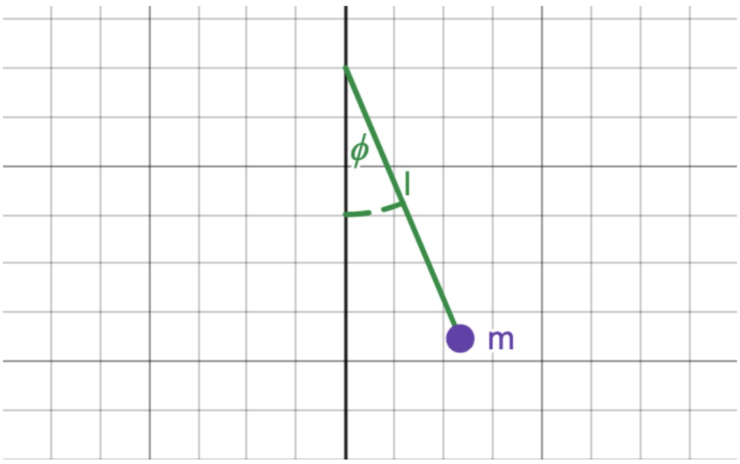
The **P-branch corresponds to transitions where the rotational quantum number decreases** (in this case from $J = 1$ to $J = 0$). These transitions typically result in **lower frequencies** because **the molecule loses rotational energy** in the process.

Subject Module	Psysics
Advanced Task 1	

Oscillations and Corrections

The description of rotational motion requires the moment of inertia I of rigid bodies, which is defined as an integral over the mass of the body: $I = \int r_{\perp}^2 \, \mathrm{d}m$, where $r_{\perp}$ is the perpendicular distance to the mass element dm in the body from the axis of rotation $\vec{\omega}$. The equation of motion, analogous to the linear case, is then $\vec{\tau} = \frac{d}{dt}\vec{L}$, where $\vec{L} = I\,\vec{\omega}$. The parallel axis theorem states that the moment of inertia of a body with respect to an axis shifted parallel to the centre-of-mass is equal to the moment of inertia with respect to its centre-of-mass plus the product of the body's mass and the shift length squared.

The pendulum bob on a string, as shown below, is one example of an oscillating system



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Question 1

A mass *m* is tied to a string of length l and experiences the net force resulting from gravity and the string. Under certain conditions, the system oscillates nearly harmonically with fixed angular frequency ω_0. This makes it possible to determine the gravitational acceleration g via $\omega_0 = \sqrt{\frac{g}{l}}$.

Which of the following is <u>not</u> one of the underlying assumptions needed for its derivation?

a) Angular displacements are small relative to the string length.
b) The mass of the string is negligible in comparison to the mass of the pendulum bob.
c) The string does not experience elastic deformations.
d) The pendulum bob is released from a stationary angular displacement.

Question 2

A first correction to the description of the simple pendulum is made by considering the mass on the string as a homogenous sphere of radius r , rather than as a point-like particle.

Which is the correct integral to calculate the moment of inertia of this body?

a) $I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin(\theta)\, dr\ d\theta\, d\phi$
b) $I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} r^4 \sin(\theta)\, dr\ d\theta\, d\phi$
c) $I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} (r^2 \sin(\theta))^2\, dr\, d\theta\, d\phi$
d) $I = \rho \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin^2(\theta)\, dr\ d\theta\, d\phi$

Question 3

To account for the moment of inertia of the pendulum mass, the distance between the suspension point of the string and the centre-of-mass of the pendulum, marked by the new length l , must be considered.

How does the moment of inertia I have to be adjusted in this improved model?

a) $I = m\left(l^2 + \frac{2}{5}r^2\right)$
b) $I = l^2 \cdot \frac{2}{5}mr^2$
c) $I = m\left(l^2 + \frac{4\pi}{3}r^3\right)$
d) $I = l^2 \cdot \frac{4\pi}{3}mr^3$

Question 4

A further correction to the model is to consider friction due to air resistance. This would contribute a force term proportional to the velocity of the body, $F_R = -2\gamma m \cdot v(t)$, where 2γ is the damping ratio. Solutions are then $x(t) = Ae^{\lambda t}$, with $\lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$.

For weak damping, $\gamma < \omega_0$, what is the new angular frequency ω ?

a) $\omega = \omega_0 - \gamma$
b) $\omega = \omega_0 + \gamma$
c) $\omega = \sqrt{\gamma^2 - \omega_0^2}$
d) $\omega = \sqrt{\omega_0^2 - \gamma^2}$

Question 5

The motions of the simple pendulum are simple harmonic oscillations characterised by the angular frequency ω_0. In addition to the air resistance and the spatial expansion of the pendulum mass, large displacements of the pendulum mass should now also be taken into account. So, there are now three corrections to the description of the system.

Which of the following correctly includes all three corrections to the angular frequency?

a) $\omega^2 = \omega_0^2 \cdot \left(1 - \frac{A^2}{8}\right) \cdot \left(1 + \frac{2}{5}\frac{r^2}{l^2}\right)^{-1} \cdot \left(1 - \frac{\gamma^2}{\omega_0^2}\right)$
b) $\omega^2 = \omega_0^2 \cdot \left(1 - \frac{A^2}{8}\right) \cdot \left(1 + \frac{4\pi}{3}\frac{r^3}{l^2}\right)^{-1} \cdot \left(1 - \frac{1}{2}\ \frac{\gamma^2}{\omega_0^2}\right)^2$
c) $\omega^2 = \omega_0^2 \cdot \left(1 - \frac{A^2}{8}\right) \cdot \left(1 + \frac{2}{5}\frac{r^2}{l^2}\right)^{-1} \cdot \left(1 - \frac{1}{2}\ \frac{\gamma^2}{\omega_0^2}\right)$
d) $\omega^2 = \omega_0^2 \cdot \left(1 - \frac{A^2}{4}\right) \cdot \left(1 + \frac{4\pi}{3}\frac{r^3}{l^2}\right)^{-1} \cdot \left(1 - \frac{1}{2}\ \frac{\gamma^2}{\omega_0^2}\right)$

Question 6

An example of an electromagnetic oscillating system is a series circuit made up of a capacitor of capacitance C , an inductor of inductance L and a resistance R . The resistance R summarises alll non-negligible resistances of the circuit. The circuit is externally driven by an alternating voltage, $U(t) = U_0 e^{i\omega t}$, and satisfies the mesh rule: $U_L + U_C + U_R = U(t)$.

Which is a true statement about this system?

a) The impedance Z is time-dependent, defined as the ratio of each component's voltage and current
b) At resonance, U_C and U_L are equal in magnitude, but opposite in phase
c) The voltage across the capacitor, U_C, leads the voltage across the inductor, U_L, by phase of $90°$.
d) At resonance, the current is minimal, and the voltage is at maximum.

Question 7

Consider two oscillating pendulum bobs on a string of equal length with a horizontal spring with the spring constant k' in between. The general equations of motion for each mass are then: $m \frac{d^2}{dt^2} x_1(t) = -kx_1(t) + k'\big(x_2(t) - x_1(t)\big)$ and $m \frac{d^2}{dt^2} x_2(t) = -kx_2(t) - k'\big(x_2(t) - x_1(t)\big)$.

If the initial conditions are $x_1(0) = A$, $x_2(0) = 0$, which statement describes the correct behaviour of the coupled oscillating system?

a) The oscillation stabilises for in-phase oscillations, meaning that for large time intervals, the amplitudes of each mass are equal in both magnitude and direction.
b) The oscillations are not in phase, resulting in a continous energy loss due to the interaction with the spring.
c) The oscillation amplitudes are modulated, meaning they become equal but opposite in direction as time t approaches infinity.
d) The potential energy of the system is initially stored in pendulum 1 and is transferred to pendulum 2 throughout the oscillations until it reaches a maximum there. This process repeats

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Subject Module	Physics
Advanced Task 1: Solutions	

Question 1

Solution: D

Since options A to C refer to the physical idealizations of the model, whereas the last option refers to an initial configuration – the initial configuration is independent of the oscillation frequency.

Question 2

Solution: B

A correct application of the transformation formula to spherical coordinates, $dm = \rho dV = \rho \cdot r^2 \sin\theta \; dr d\theta d\phi$, times the already existent distance squared combines to $r^4 \sin\theta \; dr d\theta d\phi$.

Question 3

Solution: A

Due to the new definition of the length – now from the suspension point to the centre-of-mass (cm) – such that $I = m \cdot l^2 + I_{cm}$.

Question 4

Solution: D

Due to the complex representation of the exponential: $\exp\left\{\pm\sqrt{\gamma^2 - \omega_0^2}t\right\} = \exp\left\{\pm\sqrt{-(\omega_0^2 - \gamma^2)}t\right\} = \exp\left\{\pm i\sqrt{(\omega_0^2 - \gamma^2)}t\right\} = \exp\{\pm i\omega t\}$.

Question 5

Solution: A

The first factor results from a Taylor-expansion of the sine-function, the second from incorporating the pendulum mass and the third by factoring out ω_0 from the damped oscillator.

Question 6

Solution: B

Because the capacitor voltage is phase-shifted to the inductance voltage but must be equal in maximal value.

Question 7

Solution: D

This is most simply understood by imagining the system – after the first mass is oscillating, the coupling leads to an induced oscillation of mass 2. Due to conservation of energy, this energy transfer itself oscillates between masses.

Subject Module	Physics
Advanced Task 2	

Waves and Solid States

A wave Ψ propagates through time t and space $\vec{x}$. The relationship between the wavelength λ and frequency f in vacuum is given by: $c = \lambda f$,

where c is the speed of light.

Miller indices are triples (hkl) that define a lattice plane in solid state physics. They are determined by 1. noting the numerical values of the axis intercepts, 2. taking their inverse, and 3. finding the smallest common multiple.

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Question 1

From the *Doppler effect*, the change of the wavelength of emitted sound waves from a source moving away is given by:

$$\lambda' = \lambda - v_S T \ ,$$

where T is the period of the emitter and v_S is the velocity of the moving source. The period is related to the propagation speed of the sound wave, $v_{ph} = \frac{\lambda}{T}$.

Substituting this relationship leads to the result: $\lambda' = \lambda\left(1 + \frac{v_S}{v_{ph}}\right)$.

What is the frequency f of a source moving towards the observer?

a) $f' = f\left(1 + \frac{v_S}{v_{ph}}\right)$
b) $f' = \frac{f}{\left(1+\frac{v_S}{v_{ph}}\right)}$
c) $f' = \frac{f}{\left(1-\frac{v_S}{v_{ph}}\right)}$
d) $f' = f\left(1 - \frac{v_S}{v_{ph}}\right)$

Question 2

Now consider the diffraction of light waves at a double slit of width D . The path difference Δs between neighbouring elementary waves is $\Delta s = d \sin\theta$, where d is the distance between these waves and θ the angle of opening towards the screen. By a superposition of $N = \frac{D}{d}$ such waves, the intensity distribution on the screen is:

$$I(\theta) = I_0 \frac{\sin^2\left(\frac{\pi D}{\lambda}\sin\theta\right)}{\left(\frac{\pi D}{\lambda}\sin\theta\right)^2}.$$

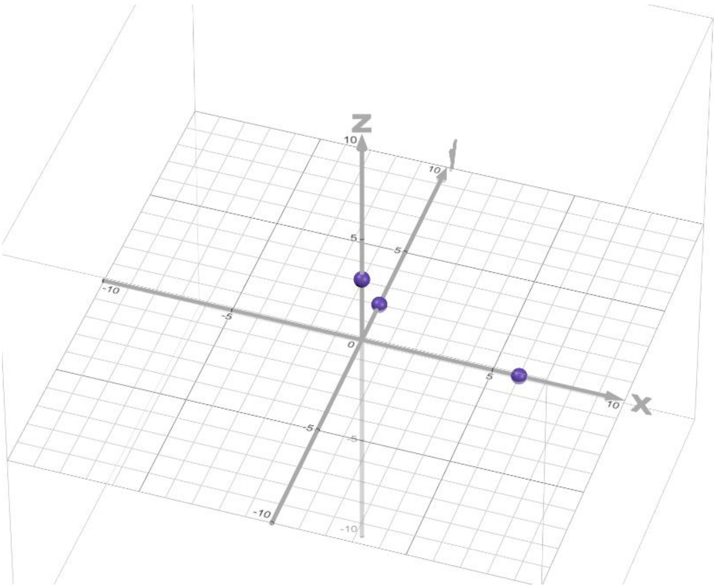
How does the diffraction pattern change if $D \ll \lambda$?

a) A single uninterrupted light ray is seen on the screen. The transition to geometric optics from wave optics is achieved.
b) Many diffraction maxima appear with additional modulation by interference patterns. The diffraction patterns are most clear here.
c) The maxima of the diffraction patterns become very wide, increasing for smaller slit widths.
d) There is no significant change.

Question 3

The diffraction pattern of *Röntgen waves* on solids allows the exploration of their structure. The condition for constructive interference is made more precise in solid state physics, by requiring $m\lambda = 2d_{hkl}\sin\theta$. hkl are the Miller indices, specifying the lattice plane.

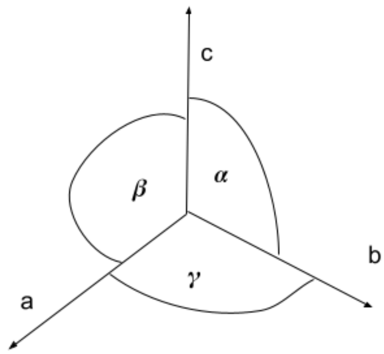
Which What are the correct Miller indices for the lattice plane with intersection coordinates $x = 6$, $y = 2$, $z = 3$ (see figure below)?



a) (6,2,3)
b) (3,3,2)
c) (1,2,3)
d) (1,3,2)

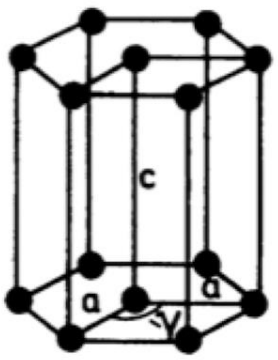
Question 4

In solid state physics, crystal systems are different structures of ideal crystalline solids. The description of the crystal's lattice is made using the translation vector $\vec{R} = l\vec{a} + n\vec{b} + m\vec{c}$, where the vectors are spanning the entire crystal. The figure below illustrates this.

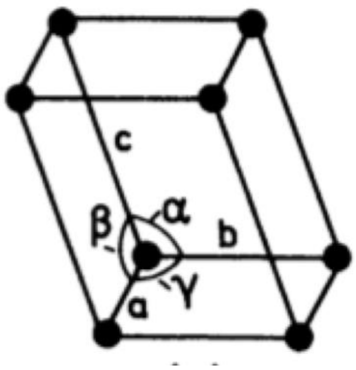


Which lattice is associated with a crystal system where $a = b \neq c$, and $\alpha = \beta = 90°$, $\gamma = 120°$?

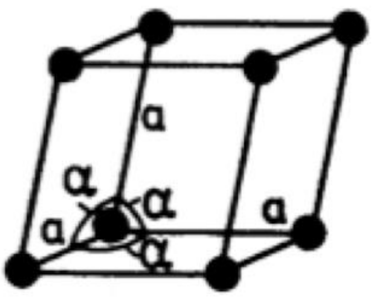
a) Hexagonal primitive



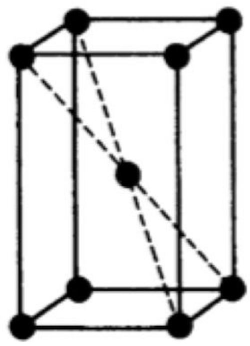
c) Trikline primitive



b) Rhombohedral primitive



d) Tetrahedral body-centered



Subject Module **Physics**

Advanced Task 2: Solutions

Question 1

Solution: C

First inverting and multiplying by c to obtain the frequency:

$$f' = \frac{c}{\lambda'} = \frac{c}{\lambda\left(1+\frac{v_S}{v_{ph}}\right)} = \frac{f}{1+\frac{v_S}{v_{ph}}},$$

then changing direction of source velocity: $v_S \rightarrow -v_S$.

Question 2

Solution: C

- The half width of the primary maximum is $\frac{\pi D}{\lambda}\sin\theta_H = \pi \Rightarrow \sin\theta_H = \frac{\lambda}{D}$
- For D much smaller than the wavelength, the sin is larger than 1, i.e. the maximum lies outside of the domain. Hence, there is one very wide pattern on the screen
- Alternatively, by physical reasoning: for D much smaller than the wavelength, there can only be a single spherical wave propagating, which produces one large interference pattern.

Question 3

Solution: D

Following steps of infobox, first inverting then finding the smallest common multiple: $(6,2,3) \rightarrow \left(\frac{1}{6},\frac{1}{2},\frac{1}{3}\right) \rightarrow (1,3,2)$

Question 4

Solution: A

Careful observation of the properties displayed in the figure – only Answer A satisfies the required relations.

Combinational Logic

Remember the boolean operation AND, OR and NOT. Their representation as logical gates in circuits is shown in Figure 1 below.

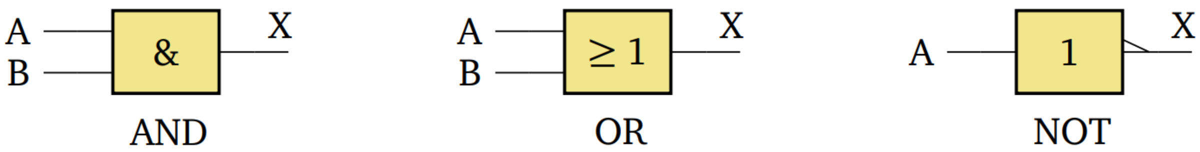


Figure 1. Logical gates in circuits of boolean operations

They are represented formally as functions $X = (A \wedge B)$ for AND, $X = (A \wedge B)$ for OR and $X = \overline{A}$ for NOT. The output X can be given by the combinations of the possible inputs. For boolean operations the input can only be 0 or 1 (sometimes also referred to as *false* and *true*). For completeness, the output of all three operations is presented using truth tables:

A	B	$A \wedge B$	$A \vee B$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

A	$\overline{A}$
0	1
1	0

Table 1. Truth table

These three boolean operations are the building blocks for more complex boolean functions or boolean circuits. As long as there are no cycles in the circuit, these circuits implement combinational logic. The resulting circuits are called combinatorial circuits.

From a truth table, we can immediately derive a boolean function or the combinatorial circuit by using what is called the *canonical disjunctive normal form* "(*CDNF*)". For each 1 that results from a function a *minterm* is created. A minterm is an AND operation over all inputs, where 0-inputs are negated while 1-inputs are not. All minterms are finally combined with an OR operation.

As an example, the CDNF of AND derived from its truth table has only one minterm $X = (A \wedge B)$. It thus does not need to be used in an OR. However, the OR operation as CDNF has three minterms that need to be combined with OR: $X = (\overline{A} \wedge B) \vee (A \wedge \overline{B}) \vee (A \wedge B)$.

Question 1

Given the following truth table, which Boolean function does the table represent?

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

a) $X = \bar{A} \wedge \overline{B}$
b) $X = \overline{A \wedge B}$
c) $X = A \vee B$
d) $X = \bar{A} \vee B$

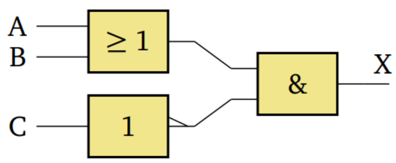
Question 2

Imagine a jewelry shop that has sensors at all of its windows, doors, and showcases. Its burglar alarm should go off if at least one of the sensors detects unusual conditions. Which Boolean operation best describes the behavior of the alarm system?

a) AND operation
b) OR operation
c) one AND combined with several OR operations
d) one OR combined with several AND operations

Question 3

The following combinatorial circuit has three inputs A, B and C The corresponding truth table has some missing entries. What are these missing values?



A	B	C	X
0	0	0	0
0	0	1	{i}
0	1	0	1
0	1	1	{ii}
1	0	0	{iii}
1	0	1	0
1	1	0	1
1	1	1	0

a) $i = 0; ii = 1; iii = 1$

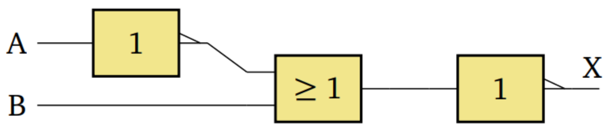
b) $i = 1; ii = 1; iii = 1$

c) $i = 0; ii = 0; iii = 0$

d) $i = 0; ii = 0; iii = 1$

Question 4

What is the correct boolean function for the shown combinatorial circuit?



a) $X = \overline{\bar{\mathrm{A}} \vee \mathrm{B}}$

b) $X = \overline{\mathrm{A} \vee \bar{\mathrm{B}}}$

c) $X = A \wedge \overline{B}$

d) $X = \overline{\bar{\mathrm{A}} \wedge \mathrm{B}}$

Question 5

What is the CDNF for the following truth table?

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

a) $X = (\bar{A} \wedge B) \vee (A \wedge \overline{B}) \vee (A \wedge B)$

b) $X = \left(\bar{A} \wedge \overline{B}\right) \vee (A \wedge B)$

c) $X = (\bar{A} \wedge B)$

d) $X = (\bar{A} \wedge B) \vee (A \wedge \overline{B})$

Question 6

You have a truth table with four inputs A, B, C and D. In one of the lines the result X is 1 for the following combination of inputs:

A	B	C	D	X
1	0	1	1	1

What is the correct minterm for this line?

a) $A \wedge \overline{B} \wedge C \wedge D$

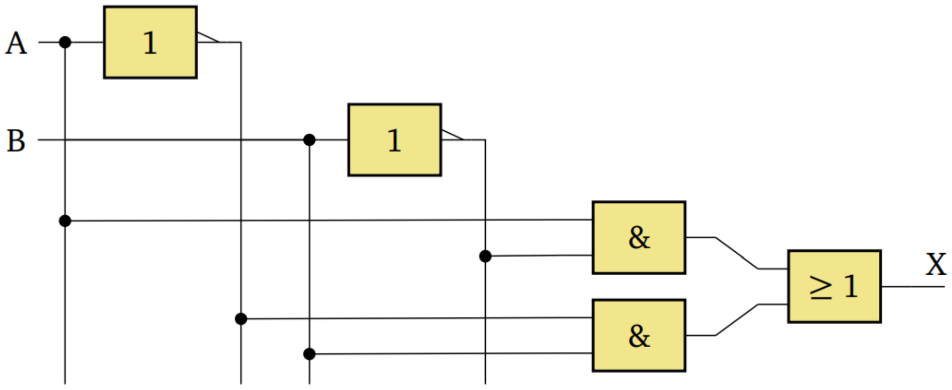
b) $A \vee \overline{B} \vee C \vee D$

c) $\overline{A} \wedge B \wedge \overline{C} \wedge \overline{D}$

d) $A \wedge C \wedge D$

Question 7

What is the truth table for the following circuit?:



a)

A	B	X
0	0	1
0	1	1
1	0	0
1	1	0

b)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

c)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

d)

A	B	X
0	0	0
0	1	0
1	0	1
1	1	1

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Subject Module	Computer Science
Advanced Task 1: Solutions	

Question 1

Solution: A

We have to look at the assignments of A and B that lead to X being evaluated to 1. There is only one assignment that evaluates to $X = 1$. Therefore, X is only 1 if the negation of A is 1 and the negation of B is 1.

Question 2

Solution: B

Each of the sensor can be represented with one variable that is either 1 (sensor detects anomaly) or 0 (sensor detects no anomaly). Since only one of the sensors needs to detect an anomaly for the alarm to go off (entire Boolean formula is 1), we put all the variables in one large OR operation.

Question 3

Solution: D

The corresponding formula for the circuit is $X = (A \vee B) \wedge \bar{C}$. We can just plug in the assignments to get X. For {i} and {ii} C is not 0. Only {iii} evaluates to 1 since both $\overline{C}$ and A are 1.

Question 4

Solution: A

We go from left to the right to derive the formula and determine the order of operations. A is negated and used as input, which is $\bar{A}$ as formula. Now we concatenate the result with B using an OR operation, which is equivalent to $\bar{A} \vee B$. Finally, the entire result X is negated, which means: $X = \overline{\bar{A} \vee B}$.

Question 5

Solution: D

To determine the **Canonical Disjunctive Normal Form (CDNF)** for the given truth table, we first identify the rows where the output $X = 1$. These rows represent the minterms that will be included in the CDNF.

From the truth table, $X = 1$ for the following input values:

- $A = 0$, $B = 1$, which corresponds to the minterm $\bar{A}B$.
- $A = 1$, $B = 0$, which corresponds to the minterm $A\overline{B}$.

The CDNF is the disjunction (OR operation) of these minterms:

$$X = (\bar{A} \wedge B) \vee (A \wedge \overline{B})$$

Question 6

Solution: A

Since in the exercise a minterm is explicitly mentioned, we have an AND operation for this line. So, we simply negate the variables that are 0 in the table (B) and combine all of the literals using an AND operation.

Question 7

Solution: C

The two AND operations are $(A \wedge \overline{B})$ for the upmost gate and $(\bar{A} \wedge B)$ for the lower one. They are combined using an OR operation, so the resulting CDNF is: $\left(A \wedge \overline{B}\right) \vee (\bar{A} \wedge B)$. Now we can just check which given truth table satisfies this CDNF.

Linear Transformations of High-Dimensional Data Sets

Linear transformations are a fundamental concept in data science. They are often used to reduce the dimensions of data sets (commonly represented as matrices). This reduction is necessary to make high-dimensional data easier to process and analyse. The basis for such transformations is *linear matrix algebra*. Fundamental concepts in matrix algebra, which are also the basis for the application of linear transformations, are the *rank*, the *dimensions*, and the *eigenvalues* of a matrix.

The rank of a matrix is its number of linearly independent rows or columns. A row or column is linearly independent if it cannot be expressed as a linear combination of the other rows or columns in the matrix. The rank of a matrix can be found using row or column operations to reduce the matrix to row-echelon form, and then counting the number of non-zero rows or columns.

The dimension of a matrix is given by its number of rows and columns. For example, a matrix with *m* rows and *n* columns has a dimension of $m \; x \; n$.

An eigenvalue is a scalar value that represents how the linear transformation stretches or shrinks a vector in a given direction. To calculate the eigenvalues of a matrix, solve the equation:

$$\det(\mathrm{A} - \lambda\mathrm{I}) = 0$$

where *A* is the matrix, *λ* is the eigenvalue, and *I* is the identity matrix of the same size as *A*. The *determinant det* of a matrix is a scalar value that indicates whether the matrix is invertible and describes the scaling factor of the linear transformation represented by the matrix. For a *2x2* matrix, A = $\binom{a\;b}{c\;d}$ the determinant is calculated as:

det(A) = ad – bc.

The resulting equation is a polynomial of degree *n*, where n is the size of the matrix. The roots of this polynomial are the eigenvalues of the matrix. Once we have found the eigenvalues, we can then find the corresponding eigenvectors by solving the equation:

$$(\mathrm{A} - \lambda\mathrm{I}) * x = 0$$

where *x* is the eigenvector corresponding to the eigenvalue *λ*.

Principal Component Analysis (*PCA*) is a common linear transformation method that is used to reduce the dimensionality of the data while retaining as much of the variation in the data as possible. The resulting lower-dimensional space is called the principal subspace. PCA can be used to identify patterns in data and to identify the most important variables for further analysis.

Another technique for linear transformation is *Linear Discriminant Analysis* (*LDA*). LDA is a supervised learning method that is used to find linear combinations of features that can best separate different classes in the data. The resulting lower-dimensional space is optimized for classification performance. LDA can be used to identify the variables that are most important for predicting a specific outcome.

Reconstruction error is a measure of how well a reduced-dimensionality representation of a data set can reconstruct the original data points. It is defined as the sum of the squared distances between each original data point and its reconstructed counterpart in the reduced-dimensionality space. In other words, it is a measure of how much information is lost when reducing the dimensionality of the data.

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Question 1

Please consider the following matrix:

| 2 1 |

| 1 2 |

What is the rank of the matrix?

a) 1
b) 2
c) 4
d) 3

Question 2

Please consider the following matrix:

| 2 1 |

| 1 2 |

What are the eigenvalues of the matrix?

a) 2, 2
b) 1, 3
c) 1, 1
d) 3, 1

Question 3

Please consider the following matrix:

| 2 1 |

| 1 2 |

If the matrix is transformed using PCA, what is the dimensionality of the resulting matrix when keeping only the first principal component?

a) 2×2
b) 1×1
c) 1×2
d) 2×1

Question 4

Please consider the following matrix:

| 2 1 |

| 1 2 |

If the matrix is transformed using LDA with two classes, what is the dimensionality of the resulting matrix when keeping only the first linear discriminant?

a) 2×1
b) 1×1
c) 1×2
d) 2×2

Question 5

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

What is the dot product of *u* and *v*?

a) 12
b) 6
c) 32
d) 56

Question 6

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following statements is true about a matrix $A \in R\, n \times n$ with *n*-dimensional column vectors a_i, if *A* is invertible?

a) A has to have full rank, which is equivalent to $a_1, \ldots, a_n$ being linearly independent and the determinant must be non-zero.
b) $a_1, \ldots, a_n$ are linearly dependent, and must <u>not</u> have a full rank.
c) The determinant of *A* has to be exactly zero, and $a_1, \ldots, a_n$ are linearly independent.
a) d) *A* has to have full rank. The determinant of *A* can be zero or non-zero.

Question 7

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following is not a commonly used linear method for dimensionality reduction?

a) They cannot handle missing values in the data set.
b) They are computationally expensive.
c) They are prone to overfitting.
d) They can only handle linear relationships between variables.

Question 8

Please consider the following two vectors:

$u = [1, 2, 3]$

$v = [4, 5, 6]$

Which of the following is a common approach to select the number of principal components to retain in PCA?

a) Retain principal components that explain at least 90% of the variance in the data set.
b) Retain all principal components.
c) Retain only the first principal component.
d) Retain principal components that explain the least amount of variance in the data set.

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Subject Module	Computer Science
Advanced Task 2: Solutions	

Question 1

Solution: B

The matrix is full rank, which can be determined by calculating the determinant $\det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 2 * 2 - 1 * 1 = 3$, which is not 0. In a full rank matrix, all rows are independent, and the rank is equal to the largest possible rank, which is 2 for this matrix.

Question 2

Solution: B

We can find the eigenvalues by solving the characteristic equation $\det(A - \lambda \mathrm{I}) = \det\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \det\begin{pmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{pmatrix} = (2 - \lambda)(2 - \lambda) - 1 * 1 = 0$

The values that solve this equation are 1 and 3.

Question 3

Solution: D

It is a general convention in data science that rows represent individual observations, while columns represent the features. The first principal component determines the direction of the maximum variance for all data points. If we apply it to the given matrix, we reduce the feature space to 1. The resulting matrix therefore has a dimensionality of $2\ x\ 1$, if we just keep the first principal component.

Question 4

Solution: A

We simply calculate the sum of the products of their corresponding components: $[1,2,3] * [4,5,6] = 1 * 4 + 2 * 5 + 3 * 6$, which is 32.

Question 5

Solution: C

If we view a matrix A with size $m \; x \; n$ as a linear transformation $T: R^n \rightarrow R^m$, with $T(x) = Ax$, the rows must be independent for the transformation to be reversible (*T* has to be bijective). If the rows are not independent, two different vectors *x* and *y* could potentially map to the same vector in R^m, which would violate the bijective property of a reversible transformation. In other words, if the rows are not independent, information is lost in the transformation, which cannot be recovered.

Question 6

Solution: A

The Random Forest Algorithm is a machine learning method for classification and regression by building decision trees and aggregating them. It is not commonly used for dimensionality reduction, since there are no transformations to the feature space directly.

Question 7

Solution: D

Linear methods for dimensionality reduction operate under the assumption that the variables have linear relationships. This, however, is not always the case, which is especially true for real-world data.

Question 8

Solution: A

In PCA, each principal component explains some of the total variance of the dataset. The first principal component captures the most variance, the second captures the second most, and so on. By using the variance as an approximated measure of information, we aim to retain 90%, while reducing the dimensionality as far as possible.

Series and Parallel Connections of Ohmic Resistors

Resistors can be connected in series, in parallel, or in any combination thereof.

Series connections

If n resistors with the resistance values $R_1, R_2, R_3, \ldots R_n$ are connected in series, the resulting total resistance R_{res} is:

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \ldots + R_{\text{n}} = \sum_{i=1}^{n} R_{\text{i}} .$$

In special cases the equation can be simplified.

In the case that all n resistors have the same resistance value R, you get:

$$R_{\text{tot}} = n \cdot R .$$

In the case of two resistors with the value ratio $R_2 = k \cdot R_1$ you can write:

$$R_{\text{tot}} = R_1 + R_2 = R_1 + k \cdot R_1 = (1+k) \cdot R_1 \quad \text{or} \quad R_{\text{tot}} = R_1 + R_2 = \frac{R_2}{k} + R_2 = \frac{k+1}{k} R_2 .$$

If the total voltage U_{tot} is applied to the series connection of the n resistors, then the following applies to the voltage U_{x} at a single resistor R_{x} (with $x = 1, 2, 3 \ldots n$):

$$U_{\text{x}} = \frac{R_{\text{x}}}{\sum_{i=1}^{n} R_{\text{i}}} U_{\text{tot}}$$

(= formula for so-called “ohmic voltage divider”).

Parallel connections

If the n resistors with the resistance values $R_1, R_2, R_3, \ldots R_n$ are connected in parallel, the following applies for the resulting total resistance R_{tot}:

$$R_{\text{tot}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_{\text{n}}}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_{\text{i}}}}$$

This formula simplifies for only two parallel resistors with the value R_1 and R_2 to:

$$R_{\text{tot}} = \frac{R_1 \cdot R_2}{R_1 + R_2} .$$

In special cases the equation can be simplified:

If all n resistors have the same resistance value R, you get:

$$R_{\text{tot}} = \frac{1}{n} R .$$

In the case of two resistors with the value ratio $R_2 = k \cdot R_1$, you can write:

$$R_{\text{tot}} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{R_1 \cdot k \cdot R_1}{R_1 + k \cdot R_1} = \frac{k}{1+k} R_1 \quad \text{or} \quad R_{\text{tot}} = \frac{R_2 / k \cdot R_2}{R_2 / k + R_2} = \frac{R_2 \cdot R_2}{(1+k) R_2} = \frac{R_2}{1+k} .$$

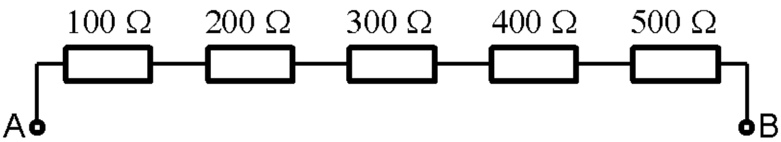
Especially if k is an integer, these formulas may simplify a calculation significantly.

If the total current I_{tot} flows through the n parallel resistors with the total resistance value R_{tot}, then the following applies to the current I_{x} through the individual resistor R_{x} (with $x = 1, 2, 3, \ldots n$):

$$I_{\text{x}} = \frac{R_{\text{tot}}}{R_{\text{X}}} I_{\text{tot}} = \frac{1}{R_{\text{X}} \cdot \sum_{i=1}^{n} \frac{1}{R_{\text{i}}}} I_{\text{tot}}$$

(= formula for “ohmic current-divider”).

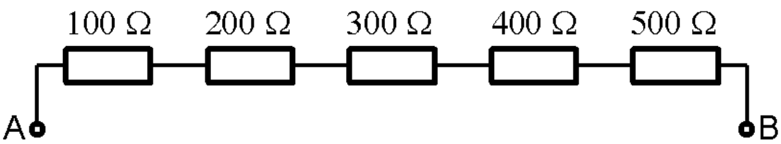
Question 1



What is the value of the total resistance between the terminals A and B?

a) $1000\,\Omega$
b) $1200\,\Omega$
c) $1400\,\Omega$
d) $1500\,\Omega$

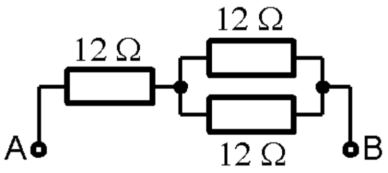
Question 2



There is the voltage $U_{\mathrm{AB}} = 500\,\mathrm{V}$ between terminals A and B.
What voltage occurs at the resistor $R = 300\,\Omega$?

a) $80\,\mathrm{V}$
b) $90\,\mathrm{V}$
c) $100\,\mathrm{V}$
d) $110\,\mathrm{V}$

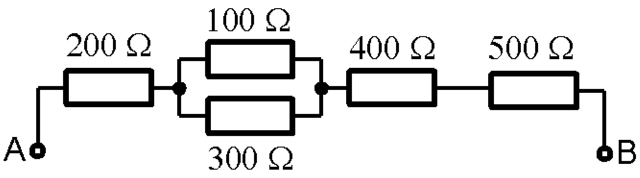
Question 3



What is the value of the total resistance between the terminals A and B?

a) 12Ω
b) 18Ω
c) 24Ω
d) none of the three named resistances

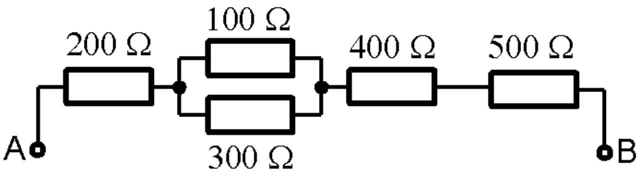
Question 4



What is the value of the total resistance between the terminals A and B?

a) 1125Ω
b) 1150Ω
c) 1175Ω
d) 1200Ω

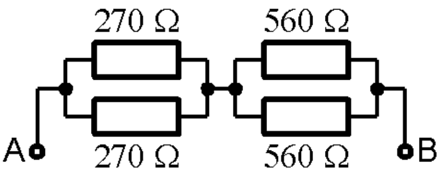
Question 5



There is the voltage $U_{AB} = 1000\,\mathrm{V}$ between terminals A and B.
On which resistor(s) is the voltage $U_R \approx 340\,\mathrm{V}$?

a) $200\,\Omega$
b) $100\,\Omega$ parallel $300\,\Omega$
c) $400\,\Omega$
d) $500\,\Omega$

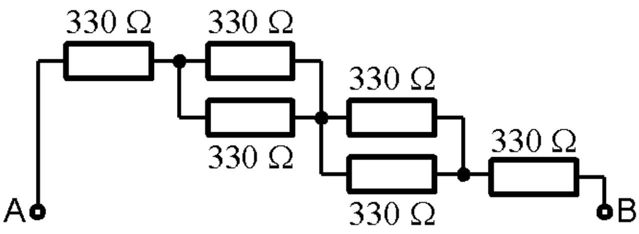
Question 6



What is the value of the total resistance between the terminals A and B?

a) $415\,\Omega$
b) $425\,\Omega$
c) $435\,\Omega$
d) $445\,\Omega$

Question 7



What is the value of the total resistance between the terminals A and B?

a) $960\,\Omega$
b) $990\,\Omega$
c) $1020\,\Omega$
d) $1030\,\Omega$

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Subject Module	Electrical Engineering
Advanced Task 1: Solutions	

Question 1

Solution: D

In the text it is described that resistors connected in series can be added, that means:

$$500\Omega + 400\Omega + 300\Omega + 200\Omega + 100\Omega = 1500\Omega.$$

Question 2

Solution: C

The formula

$$U_x = \frac{R_x}{\sum_{i=1}^{n} R_i} U_{tot}$$

shows that the ratio between a resistor R_x and the total resistance corresponds to the ratio between the voltage at resistor R_x and the voltage between terminals A and B. Substituting the resistance $R = 300\Omega$, relative to the total resistance $\sum_{i=1}^{n} R_i = 1500\Omega$, times the total voltage $U_{tot} = 500V$ gives:

$$U_x = \frac{300\Omega}{1500\Omega} \times 500V = \frac{1\Omega}{5\Omega} \times 500V = 100V.$$

Question 3

Solution: B

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies:

$$R_{\text{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 12\Omega$ we get:

$$\frac{1}{2} \times 12\Omega = 6\Omega.$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$12\Omega + 6\Omega = 18\Omega.$$

Question 4

Solution: C

To find the solution, please note that two resistors are connected in parallel here. Since they do not have the same value, the following applies to them:

$$R_{\text{tot}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

By inserting $R_1 = 100\Omega$ and $R_2 = 300\Omega$ we get:

$$R_{tot} = \frac{1}{\frac{1}{100\Omega} + \frac{1}{300\Omega}}$$

Adding the two fractions in the denominator (by bringing them to the same denominator) gives:

$$R_{tot} = \frac{1}{\frac{3}{300\Omega} + \frac{1}{300\Omega}} = \frac{1}{\frac{4}{300\Omega}}$$

By forming the reciprocal you get:

$$R_{tot} = 1 \times \frac{300\Omega}{4} = 75\Omega$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$75\Omega + 200\Omega + 400\Omega + 500\Omega = 1175\Omega.$$

Question 5

Solution: C

The formula

$$U_{\mathrm{x}} = \frac{R_{\mathrm{x}}}{\sum_{i=1}^{n} R_{\mathrm{i}}} U_{\mathrm{tot}}$$

shows that the ratio between a resistor R_x and the total resistance corresponds to the ratio between the voltage at resistor R_x and the voltage between terminals A and B. The required resistance of R_x is therefore $0{,}34 \times$ total resistance (1175Ω). As an approximation, $\frac{1}{3}$ can be used, which quickly makes it clear that only 400Ω can be considered as a solution (the exact value is $399{,}5\Omega$).

Question 6

Solution: A

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies to them:

$$R_{\mathrm{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 270\Omega$, we obtain for the first parallel connection:

$$\frac{1}{2} \times 270\Omega = 135\Omega.$$

For the second parallel connection, the following is obtained by inserting $n = 2$ und $R = 560\Omega$:

$$\frac{1}{2} \times 560\Omega = 280\Omega.$$

According to the formula for series connection, the resistances can now be added to calculate the total resistance:

$$135\Omega + 280\Omega = 415\Omega.$$

Question 7

Solution: B

To find the solution, please note that two resistors are connected in parallel here. Since they have the same value, the following applies to them:

$$R_{\text{tot}} = \frac{1}{n} R$$

By inserting $n = 2$ and $R = 330\Omega$, the result is for both parallel circuits::

$$\frac{1}{2} \times 330\Omega = 165\Omega.$$

According to the formula for series connection, all resistors can now be added to calculate the total resistance:

$$330\Omega + 165\Omega + 165\Omega + 330\Omega = 990\Omega.$$

System Analysis

Control engineering deals with dynamic systems (plants). Dynamic systems can be influenced via a manipulated variable $u(t)$ and have an output signal $y(t)$, which should take on a desired value w. *Figure 1* shows this relationship in the form of the standard control loop. A disturbance $d(t)$ normally acts on the system output.

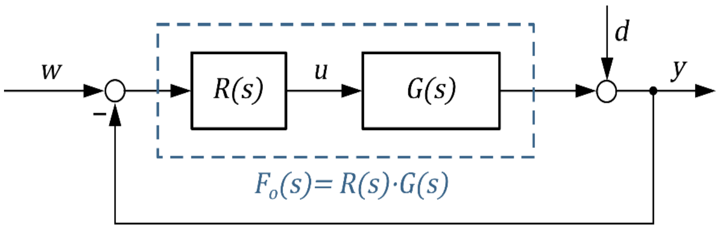


Figure 1: closed loop system – standard control loop

First, the system behavior must be identified and $G(s)$ must be determined so that a controller can be designed that fits the system. To do this, the differential equation describing the system behavior has to be set up first. The differential equation transformed into the Laplace domain provides the system transfer function $G(s)$.

The system transfer function $G(s)$ and a matching function $R(s)$ can be summarized as $F_o(s) = R(s) \cdot G(s)$ (Fig. 1).

$F_o(s)$ denotes the open control loop (index o for open loop) and is a rational function: $F_o(s) = \frac{Z_o(s)}{N_o(s)}$. $Z_o(s)$ denotes the numerator polynomial and $N_o(s)$ the denominator polynomial of $F_o(s)$. The location of the zeros of the numerator polynomial $Z_o(s)$ and the location of the poles (zeros of the denominator polynomial $N_o(s)$) provide information about important properties and the system behavior of the closed loop control system. The most important properties for control engineering are stability as well as steady state accuracy.

The *Nyquist plot* and the *root locus plot* are graphical analysis methods that describe the system behavior via a diagram in the complex number plane. In the Nyquist plot, the transfer behavior of $F_o(s)$ on the imaginary axis is considered for frequencies from $\omega = 0$ to $\omega \to +\infty$, where $s = j\omega$. The graph starts its course at $\omega = 0$.

The root locus plot takes as starting point the positions of the poles and zeros of the open loop $F_o(s)$. The root locus plot shows the position of the poles of the closed loop, if a pure proportional controller (P-controller) $R(s) = K$ is used and K is varied. The root locus plot starts its course in the poles of the open loop.

Question 1

What is the system transfer function $G(s)$ to the differential equation of the form $m \cdot \ddot{y} + c \cdot y - u = 0$?

a) $G(s) = \frac{1}{m \cdot s^2 + c}$
b) $G(s) = \frac{1}{s^2 + m \cdot s + c}$
c) $G(s) = \frac{1}{c \cdot s^2 + m}$
d) $G(s) = \frac{s}{s^2 + m \cdot s + c}$

Question 2

What is the expression for the reference transfer function $F_w(s) = \frac{Y(s)}{W(s)}$, which describes the relationship between the output signal y and the reference signal w in the closed control loop via $F_o(s)$?

a) $F_w(s) = F_o(s)$
b) $F_w(s) = F_o(s) \cdot F_o(s)$
c) $F_w(s) = \frac{F_o(s)}{1+F_o(s)}$
d) $F_w(s) = \frac{1}{F_o(s)}$

Question 3

What is the expression for the disturbance transfer function $F_d(s) = \frac{Y(s)}{D(s)}$), which describes the relationship between the output signal y and the disturbance signal d in the closed control loop via $F_o(s)$?

a) $F_d(s) = \frac{1}{1+F_o(s)}$
b) $F_d(s) = F_o(s)$
c) $F_d(s) = F_o(s) \cdot F_o(s)$
d) $F_d(s) = \frac{1}{F_o(s)}$

Question 4

Which of the following statements about the reference transfer function and the disturbance transfer function is correct?

a) In a standard control loop, if the reference transfer function has steady state accuracy, the disturbance transfer function has steady state accuracy as well.
b) In a standard control loop, if the reference transfer function is stable, the disturbance transfer function is always stable as well.
c) In the standard control loop, the reference transfer function is equal to the disturbance transfer function.
d) In the standard control loop, the reference transfer function is always stable, while the disturbance function is always unstable.

Question 5

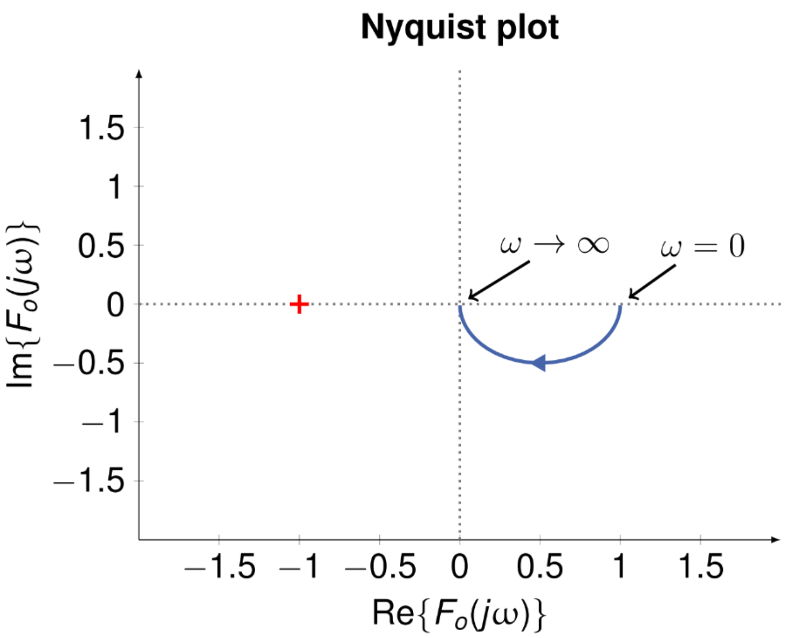


Figure 2: Nyquist plot of an unknown system

What is the transfer function $G(s)$ of the Nyquist plot shown in *Figure 2*?

a) $F_0(s) = \frac{1}{s} = F_0(j\omega) = \frac{1}{j\omega}$
b) $F_0(s) = s = F_0(j\omega) = j\omega$
c) $F_0(s) = \frac{1}{s+10} = F_0(j\omega) = \frac{1}{j\omega+10}$
d) $F_0(s) = \frac{1}{s+1} = F_0(j\omega) = \frac{1}{j\omega+1}$

Question 6

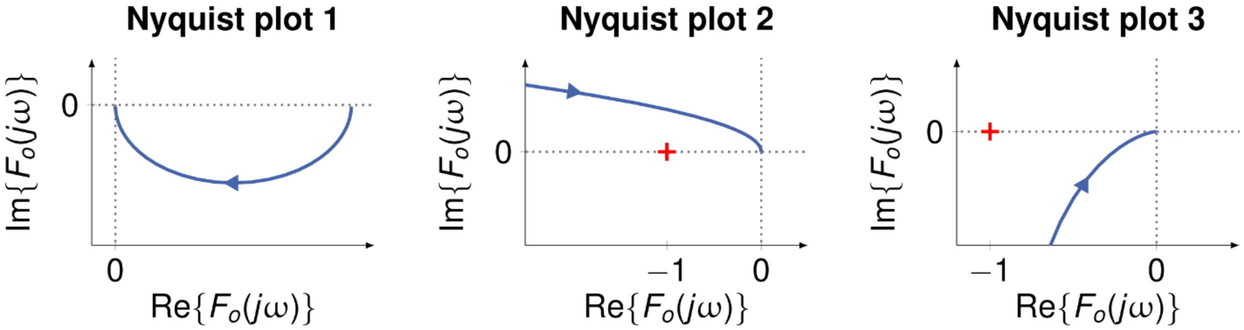


Figure 3: Nyquist plot of three systems 1, 2 and 3

What steady-state behavior is exhibited by the systems whose Nyquist plots are shown in *Figure 3*?

a) System 1: P-behavior, system 2: I^2-behavior, system 3: I-behavior
b) System 1: I^3-behavior, system 2: I^2-behavior, system 3: I-behavior
c) System 1: P-behavior, system 2: P-behavior, system 3: P-behavior
d) System 1: I-behavior, system 2: P-behavior, system 3: I^2-behavior

Question 7

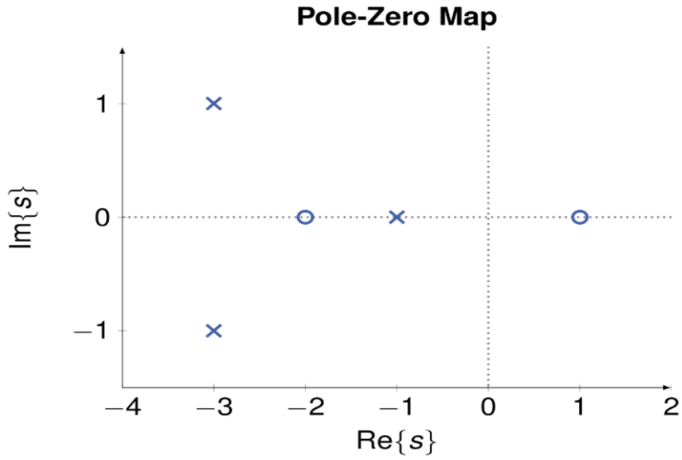


Figure 4: Pole-zero diagram of an unknown system $G(s)$

What is the system function $G(s)$ of the pole-zero diagram shown in *Figure 4*?

a) $G(s) = \frac{(s-1)(s+2)}{(s^2+6s+10)(s+1)}$

b) $G(s) = \frac{(s+1)(s-2)}{(s^2+6s+10)(s+1)}$

c) $G(s) = \frac{(s+1)(s-2)}{(s^2+6s+10)(s-1)}$

d) $G(s) = \frac{s-1}{(s^2+6s+10)(s+1)}$

Question 1

Solution: A

One describes as usual the Laplace transform

$$\mathcal{L}\{u\}(s) = \int_0^\infty u(t)e^{-st}dt$$

of the function $u(t)$ with $U(s)$ and the Laplace transform $\mathcal{L}\{y\}(s)$ of the function $y(t)$ with $Y(s)$. The Laplace transform of the second derivative $\ddot{y}$ of y is then $\mathcal{L}\{\ddot{y}\}(s) = s^2Y(s)$. The differential equation

$$m \cdot \ddot{y}(t) + c \cdot y(t) - u(t) = 0$$

by applying the Laplace transformation thus goes into the algebraic equation

$$m \cdot s^2 \cdot Y(s) + c \cdot Y(s) - U(s) = 0.$$

By transforming you obtain

$$(m \cdot s^2 + c) \cdot Y(s) = U(s)$$

or

$$Y(s) = \frac{1}{m \cdot s^2 + c} U(s).$$

The relationship between $U(s)$ and $Y(s)$ is given by $Y(s) = G(s)U(s)$, from which

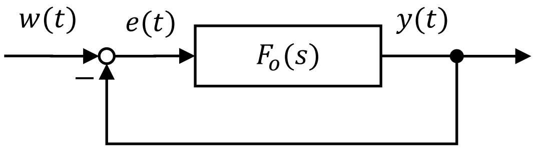
$$G(s) = \frac{1}{m \cdot s^2 + c}$$

follows.

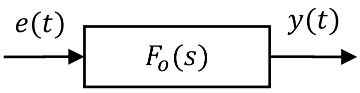
Question 2

Solution: C

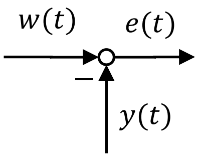
By simplifying the given block diagram and introducing the auxiliary function $e(t)$, you obtain the block diagram



By transition to the Laplace transforms you obtain for the transmission path



the equation $Y(s) = F_o(s)E(s)$ and for the node



the equation $E(s) = W(s) - Y(s)$. Thus the following applies

$$Y(s) = F_o(s)E(s) = F_o(s)\big(W(s) - Y(s)\big).$$

By transforming you obtain

$$Y(s) = F_o(s)W(s) - F_o(s)Y(s)$$

$$\Leftrightarrow Y(s) + F_o(s)Y(s) = F_o(s)W(s)$$

$$\Leftrightarrow (1 + F_o(s))Y(s) = F_o(s)W(s)$$

$$\Leftrightarrow Y(s) = \frac{F_o(s)}{1 + F_o(s)}W(s)$$

$$\Leftrightarrow \frac{Y(s)}{W(s)} = \frac{F_o(s)}{1 + F_o(s)}$$

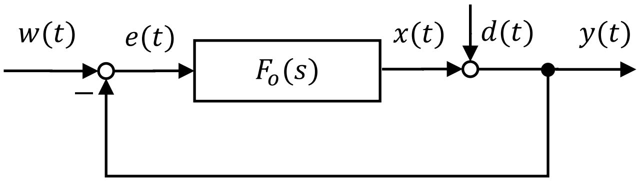
So that means

$$F_w(s) = \frac{F_o(s)}{1 + F_o(s)}$$

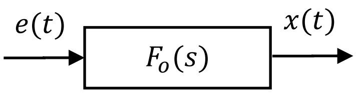
Question 3

Solution: A

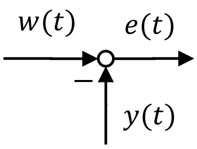
By simplifying the given block diagram and introducing the auxiliary functions $e(t)$ and $x(t)$ you obtain the block diagram



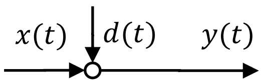
By transition to the Laplace transforms you obtain for the transmission path



the equation $X(s) = F_o(s)E(s)$. For the node



you get the equation $E(s) = W(s) - Y(s)$, and for the node



you get the equation $Y(s) = X(s) + D(s)$. Thus the following applies

$$Y(s) = X(s) + D(s) = F_o(s)E(s) + D(s) = F_o(s)\big(W(s) - Y(s)\big) + D(s).$$

Since this equation must apply independently of the reference variable $w(t)$, we consider the equation in the following for the case $w(t) = 0$. We then obtain

$$Y(s) = -F_o(s)Y(s) + D(s)$$

$$\Leftrightarrow Y(s) + F_o(s)Y(s) = D(s)$$

$$\Leftrightarrow (1 + F_o(s))Y(s) = D(s)$$

$$\Leftrightarrow Y(s) = \frac{1}{1 + F_o(s)}D(s)$$

$$\Leftrightarrow \frac{Y(s)}{D(s)} = \frac{1}{1 + F_o(s)}$$

So that means

$$F_d(s) = \frac{1}{1 + F_o(s)}$$

Question 4

Solution: B

We have shown that the command transfer function $F_w(s)$ is given by

$$F_w(s) = \frac{F_o(s)}{1 + F_o(s)}$$

and the disturbance transfer function is given by

$$F_d(s) = \frac{1}{1 + F_o(s)}$$

With the approach

$$F_o(s) = \frac{Z_o(s)}{N_o(s)}$$

you obtain

$$F_w(s) = \frac{Z_o(s)}{Z_o(s) + N_o(s)}$$

and

$$F_d(s) = \frac{N_o(s)}{Z_o(s) + N_o(s)}$$

where $Z_o(s)$ and $N_o(s)$ are polynomials. The function $Z_o(s) + N_o(s)$ is then also a polynomial. The function $F_w(s)$ is stable exactly when the real parts of all its pole points are negative. The pole places are the zeros of the polynomial $Z_o(s) + N_o(s)$, which are not at the same time zeros of the polynomial $Z_o(s)$. If you now assume that the function $F_d(s)$ is not stable, it follows that there is at least one zero λ of the polynomial $Z_o(s) + N_o(s)$ whose real part is non-negative and which is not at the same time a zero of the polynomial $N_o(s)$. Since according to the prerequisite $Z_o(\lambda) + N_o(\lambda) = 0$, λ cannot be a zero of the polynomial $Z_o(\lambda)$. Thus λ would be a pole point of $F_w(s)$, which contradicts the fact that $F_w(s)$ is stable. Thus, from the stability of $F_w(s)$ always follows the stability of $F_d(s)$.

Question 5

Solution: D

We are looking for the transfer function that yields the value 1 for $\omega = 0$, whose imaginary part is negative for $\omega > 0$ and which tends towards 0 for $\omega \to \infty$. By conjugate complex expansion you obtain for the following representation for the transfer function in d):

$$F_o(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega + 1} = \frac{1 - \mathrm{j}\omega}{(1 + \mathrm{j}\omega)(1 - \mathrm{j}\omega)} = \frac{1 - \mathrm{j}\omega}{1 + \omega^2} = \frac{1}{1 + \omega^2} + \mathrm{j}\frac{-\omega}{1 + \omega^2}$$

So that means

$$\mathrm{Re}\{F_o(\mathrm{j}\omega)\} = \frac{1}{1 + \omega^2},$$

$$\mathrm{Im}\{F_o(\mathrm{j}\omega)\} = \frac{-\omega}{1 + \omega^2}.$$

For $\omega = 0$ you obtain $\mathrm{Re}\{F_o(0)\} = 1$ and $\mathrm{Im}\{F_o(0)\} = 0$. For $\omega > 0$ the real part of $F_o(\mathrm{j}\omega)$ takes positive values between 0 and 1, and the imaginary part of $F_o(\mathrm{j}\omega)$ takes negative values. Furthermore

$$\lim_{\omega \to \infty} \mathrm{Re}\{F_o(\mathrm{j}\omega)\} = 0$$

and

$$\lim_{\omega \to \infty} \mathrm{Im}\{F_o(\mathrm{j}\omega)\} = 0$$

Therefore, the Nyquist plot shown in the graph fits the transfer function

$$F_o(\mathrm{j}\omega) = \frac{1}{\mathrm{j}\omega + 1}.$$

Question 6

Solution: B

We consider a transmission system of the form

$$\xrightarrow{e(t)} \boxed{F_o(s)} \xrightarrow{y(t)}$$

with input signal $e(t)$ and output signal $y(t)$. Then $Y(s) = F_o(s)E(s)$ applies.

The transmission system shows a P-behaviour if a differential equation of the form

$$y(t) + T\dot{y}(t) = Ke(t)$$

with positive, real constants K and T. By applying the Laplace transformation you obtain

$$Y(s)(1 + sT) = KE(s)$$

and thus

$$F_o(s) = \frac{K}{1 + Ts}.$$

For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{1 + \mathrm{j}T\omega} = \frac{K(1 - \mathrm{j}T\omega)}{(1 + \mathrm{j}T\omega)(1 - \mathrm{j}T\omega)} = \frac{K}{1 + T^2\omega^2} + \mathrm{j}\frac{KT\omega}{1 + T^2\omega^2}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the fourth quadrant of the complex plane, whereby it begins for $\omega = 0$ at the point K for $\omega \to \infty$ towards 0. The Nyquist plot for system 1 takes such a course.

The transmission system shows I-behaviour when an integral differential equation of the form

$$y(t) + T\dot{y}(t) = K\int_0^t e(\tau)\,\mathrm{d}\tau$$

with positive real constants K and T holds. By applying the Laplace transformation, you obtain

$$Y(s)(1 + sT) = \frac{K}{s}E(s)$$

and thus

$$F_o(s) = \frac{K}{\mathrm{s} + Ts^2}.$$

For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{\mathrm{j}\omega - T\omega^2} = \frac{K(-\mathrm{j}\omega - T\omega^2)}{(\mathrm{j}\omega - T\omega^2)(-\mathrm{j}\omega - T\omega^2)} = \frac{-KT\omega^2}{\omega^2 + T^2\omega^4} + \mathrm{j}\frac{-K\omega}{\omega^2 + T^2\omega^4}$$
$$= \frac{-KT}{1 + T^2\omega^2} + \mathrm{j}\frac{-K}{\omega + T^2\omega^3}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the third quadrant of the complex plane, whereby it tends towards 0 for $\omega \to \infty$. The Nyquist plot takes such a course for system 3.

An I²-behaviour exists if the transfer function of the system is of the form

$$F_o(s) = \frac{K}{s^2 + Ts^3}$$

with positive, real constants K and T. For $s = \mathrm{j}\omega$ you obtain by conjugate complex expansion the representation

$$F_o(\mathrm{j}\omega) = \frac{K}{-\omega^2 - \mathrm{j}T\omega^3} = \frac{K(-\omega^2 + \mathrm{j}T\omega^3)}{(-\omega^2 - \mathrm{j}T\omega^3)(-\omega^2 + \mathrm{j}T\omega^3)} = \frac{-KT\omega^2}{\omega^4 + T^2\omega^6} + \mathrm{j}\frac{KT\omega^3}{\omega^4 + T^2\omega^6}$$
$$= \frac{-KT}{\omega^2 + T^2\omega^4} + \mathrm{j}\frac{KT}{\omega + T^2\omega^3}.$$

The Nyquist plot of such a transfer function runs for $\omega > 0$ in the second quadrant of the complex plane, whereby it tends towards 0 for $\omega \to \infty$. This is the course of the Nyquist plot for system 2.

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Question 7

Solution: A

We are looking for the fractional rational transfer function $G(s)$, that has the zeros $z_1 = 1$ und $z_2 = -2$ and the poles $p_1 = -3 + \mathrm{j}$, $p_2 = -3 - \mathrm{j}$ and $p_3 = -1$. According to the fundamental theorem of algebra, such a function is of the form

$$G(s) = \frac{(s - z_1)^{\mu_1}(s - z_2)^{\mu_2}}{(s - p_1)^{\nu_1}(s - p_2)^{\nu_2}(s - p_3)^{\nu_3}} = \frac{(s - 1)^{\mu_1}(s + 2)^{\mu_2}}{(s + (3 - \mathrm{j}))^{\nu_1}(s + (3 + \mathrm{j}))^{\nu_2}(s + 1)^{\nu_3}}.$$

Among the given transfer functions, therefore, only the one mentioned in a) comes into question and that with $\mu_1 = \mu_2 = \nu_1 = \nu_2 = \nu_3 = 1$. In fact, the following applies

$$\begin{aligned}(s + (3 - \mathrm{j}))^1(s + (3 + \mathrm{j}))^1 &= (s + (3 - \mathrm{j}))(s + (3 + \mathrm{j})) \\ &= s^2 + (3 - \mathrm{j})s + (3 + \mathrm{j})s + (3 - \mathrm{j})(3 + \mathrm{j}) = s^2 + 3s - \mathrm{j}s + 3s + \mathrm{j}s + 3^2 - \mathrm{j}^2 \\ &= s^2 + 6s + 10\end{aligned}$$

and thus

$$G(s) = \frac{(s - 1)^1(s + 2)^1}{(s + (3 - \mathrm{j}))^1(s + (3 + \mathrm{j}))^1(s + 1)^1} = \frac{(s - 1)(s + 2)}{(s^2 + 6s + 10)(s + 1)}.$$

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Subject Module	Mechanical Engineering
Advanced Task 1	

Mathematics II

Consider the following function $f: [-1, 8] \to \mathbb{R}$ with $f(x) = \begin{cases} 1 + \sqrt[3]{x} & , x \geq 0 \\ 1 - \sqrt[3]{-x} & , x < 0 \end{cases}$.

Additionally, you are given the following calculations and notes:

The function f is differentiable for $x > 0$ with $f'(x) = \frac{1}{3} * x^{-\frac{2}{3}} = \frac{1}{3 * \sqrt[3]{x^2}}$

The function f is differentiable for $x < 0$ with $f'(x) = \frac{1}{3} * (-x)^{-\frac{2}{3}} = \frac{1}{3 * \sqrt[3]{(-x)^2}} = \frac{1}{3 * \sqrt[3]{(-x)^2}}$

The limes limits for $x = 0$ for the derivative of f are given by:

$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \infty$ and $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \infty$

Question 1

Which of the following statements is correct in respect to the differentiability of the described function?

a) The function f is differentiable for all $x \in \mathbb{R}$
b) The function f is differentiable for all $x \in (-1, 8)$
c) The function f is differentiable for all $x \in [-1, 8]$
d) The function f is not differentiable for $x = 0$

Question 2

Which of the following statements is correct in respect to the monotonicity for $x \in (-1, 0)$ of the described function?

a) The function f is monotonously increasing for $x \in (-1, 8)$
b) The function f is monotonously decreasing for $x \in (-1, 0)$
c) The function f is monotonously increasing for $x \in (-1, 0)$
d) The function f is monotonously decreasing for $x \in (-1, 8)$

Question 3

Which of the following statements is correct in respect to the global maximum of the described function?

a) The function f has no global maximum.
b) The function f has one global maximum at $x = 8$
c) The function f has one global maximum at $x = -1$
d) The function f has one global maximum at $x = 0$

Question 4

Which of the following statements is correct in respect to the global minimum of the described function?

a) The function f has no global minimum.
b) The function f has one global minimum at $x = -1$
c) The function f has one global minimum at $x = 8$
d) The function f has one global minimum at $x = 0$

Question 5

Which of the following statements is correct in respect to the monotonicity for $x \in (0, 8)$ of the described function?

a) The function f is monotonously increasing for $x \in \mathbb{R}$
b) The function f is monotonously decreasing for $x \in (0, 8)$
c) The function f is monotonously increasing for $x \in (0, 8)$
d) The function f is monotonously decreasing for $x \in \mathbb{R}$

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Question 1

Solution: D

The function f is not differentiable for $x = 0$ because both limes limits for $x = 0$ for the derivative of f are infinite.

Question 2

Solution: C

The function f is monotonously increasing for $x \in (-1, 0)$ because $f'(x) > 0$ for $x \in (-1, 0) \cup (0, 8)$ and f is not differentiable for $x = 0$.

Question 3

Solution: B

The function f has one global maximum at $x = 8$ because f is a continuous function and monotonously increasing for $x \in (-1, 0) \cup (0, 8)$.

Question 4

Solution: B

The function f has one global minimum at $x = -1$ because f is a continuous function and monotonously increasing for $x \in (-1, 0) \cup (0, 8)$.

Question 5

Solution: C

The function f is monotonously increasing for $x \in (0, 8)$ because $f'(x) > 0$ for $x \in (-1, 0) \cup (0, 8)$ and f is not differentiable for $x = 0$.

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Advanced Task 2	

The Rankine Cycle

Thermodynamic cycles are sequences of processes used in engineering systems to convert energy between heat and work. Among these cycles, the Rankine cycle is a process widely used in power plants to convert heat into mechanical work and ultimately into electricity. It utilizes a working fluid, typically water, which alternates between liquid and vapor phases as it circulates through the cycle. This specific cycle includes key components such as a boiler, turbine, condenser, and pump. In the boiler, liquid water is heated at high pressure, transforming into high-temperature steam. This steam expands through the turbine, generating mechanical work. Afterwards, the steam is cooled in the condenser, returning to liquid form, and then pressurized by the pump to restart the cycle. The diagram below illustrates the key stages of the Rankine cycle, showing the flow of the working fluid through the system and the associated processes:

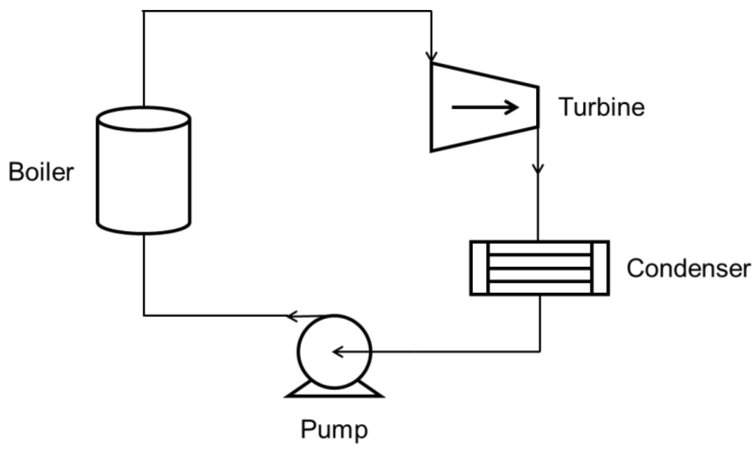


Figure 2. Rankine Cycle Illustration

The cycle's efficiency depends on the temperatures of the heat source (in the boiler) and the heat sink (in the condenser). By optimizing these temperatures, the performance of the cycle can be improved.

Question 1

Based on the T-S diagram of a vapor power cycle shown below: Which processes are included in the Rankine cycle described?

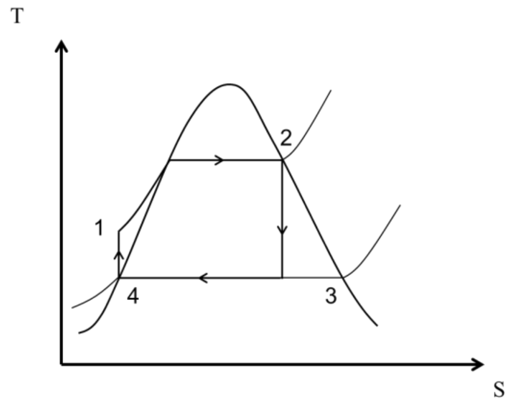


Figure. T = Temperature; S = Entropy

a) two isothermal and two isochoric processes
b) two isentropic and two isobaric processes
c) two isentropic and two isothermal processes
d) two isothermal and two isobaric processes

Question 2

What is the condition of the steam at the starting of turbine expansion?

a) wet with a dryness fraction of 0.8
b) wet with a dryness fraction of 0.99
c) dry saturated
d) superheated

Question 3

The efficiency η of the Rankine cycle based on enthalpy h is equal to…

a) $\frac{(\mathrm{h}_2-h_3)+(h_1-h_4)}{(h_2-h_1)}$

b) $\frac{(\mathrm{h}_2-h_3)-(h_1-h_4)}{(h_2-h_1)}$

c) $\frac{(\mathrm{h}_2-h_3)}{(h_2-h_1)}$

d) $\frac{(\mathrm{h}_2-h_1)}{(h_2-h_3)}$

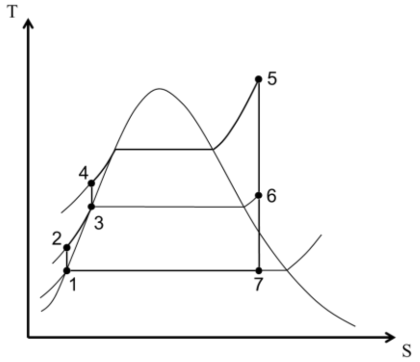
Question 4

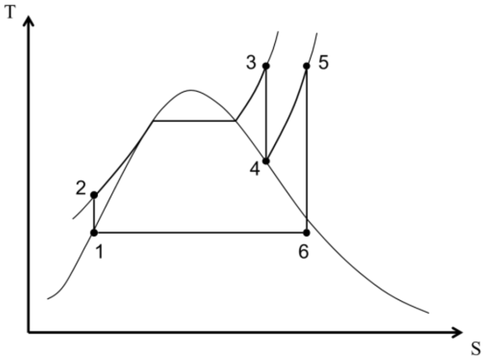
In an ideal Rankine cycle, if the condenser pressure is reduced while maintaining the same boiler pressure, how does this change affect the cycle efficiency and the turbine work output?

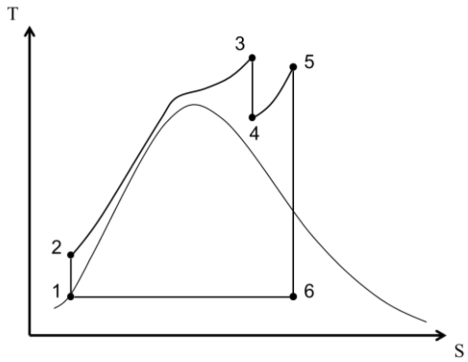
a) Both the cycle efficiency and the turbine work output increase.
b) The turbine work output increases, but the cycle efficiency decreases.
c) Both the cycle efficiency and the turbine work output decrease.
d) The turbine work output increases, and the cycle efficiency also increases.

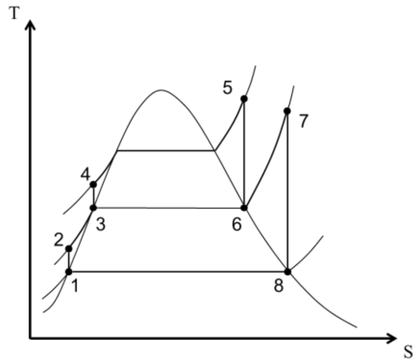
Question 5

The Rankine cycle can be modified to improve efficiency by reheating the steam after partial expansion in the turbine. This ensures that the steam expands again at a higher temperature, reducing moisture content and improving overall efficiency. Which of the following T-s diagrams represents the Rankine cycle with this modification?

a)


b)


c)


d)


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Question 1

Solution: B

The Rankine cycle consists of the following processes:

Process 1-2: Isentropic compression in a pump. This is a vertical line (constant entropy, isentropic) on the T-S diagram.

Process 2-3: Isobaric heat addition in the boiler. This is a horizontal line (constant pressure, isobaric) on the T-S diagram.

Process 3-4: Isentropic expansion in a turbine. This is a vertical line (constant entropy, isentropic) on the T-S diagram.

Process 4-1: Isobaric heat rejection in the condenser. This is a horizontal line (constant pressure, isobaric) on the T-S diagram.

Question 2

Solution: C

Process 2-3 in the T-S diagram represents the isentropic expansion of steam in the turbine.

At the start of turbine expansion (point 2 on the diagram), the steam is located at the boundary of the saturation dome.

This means the steam is dry saturated at the start of the turbine expansion, as it is on the saturation curve at the top of the dome.

If the steam were superheated, it would lie outside the saturation dome (to the right).

If the steam were wet, it would be inside the saturation dome (between the saturated liquid and saturated vapor lines).

Question 3

Solution: B

The thermal efficiency η of the Rankine cycle is given by: $\eta = \frac{\text{Net Work Output}}{\text{Heat Input}}$

Step 1: Heat Input

The heat input occurs during the isobaric process in the boiler (Process 1-2). The heat added is: $h_2 - h_1$

Step 2: Net Work Output

The net work output is the difference between:

- Work done by the turbine (Process 2-3): $h_2 - h_3$

- Work required by the pump (Process 4-1): $h_1 - h_4$

So, the net work output is: $(h_2 - h_3) - (h_1 - h_4)$

Step 3: Efficiency Formula

The efficiency is: $\eta = \frac{\text{Net Work Output}}{\text{Heat Input}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$

Question 4

Solution: D

Effect on Turbine Work Output: Lowering the condenser pressure reduces the pressure at the turbine's exit, allowing the steam to expand further. This increases the amount of work the turbine can produce.

Effect on Cycle Efficiency: A lower condenser pressure decreases the amount of heat rejected in the condenser. This increases the net work output of the cycle, improving its efficiency.

Question 5

Solution: B

In the reheated Rankine cycle:

1. Steam expands partially in the turbine (high-pressure expansion).
2. It is reheated at constant pressure to a higher temperature.
3. The steam then undergoes a second expansion in the turbine (low-pressure expansion).
4. This improves efficiency and reduces the moisture content at the turbine's final stage.

Key Characteristics of the Correct Diagram:

- Reheating must occur at constant pressure (horizontal line on the T-S diagram).
- The two expansion stages (before and after reheating) should occur with entropy remaining constant (vertical lines).

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