dMAT
Digitaler Mastertest

dMAT – Digital Master Assessment Test
General Academic Module

Preparatory Materials for Test Takers

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Imprint

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Instructions for the use of the preparation materials for test takers

Dear dMAT participant,

these preparation materials will help you to prepare well for the dMAT exam. Here you get

- general information on the content and structure of the test,
- detailed instructions and hints on how to work through the different task types as well as
- the possibility to work on exercises for each task type in different levels of difficulty including sample solutions.

Read all the information carefully to become well acquainted with the dMAT. The preparation materials are primarily intended to help you to prepare for the exam in terms of content. You can get hints and examples on the digital form of the test in information videos on www.d-mat.de.

Note

The detailed instructions for the individual task types are only available in these preparation materials! In the dMAT exam you will only see short explanations of the processing as a reminder.

We wish you lots of success!

Your dMAT team

General information about the dMAT

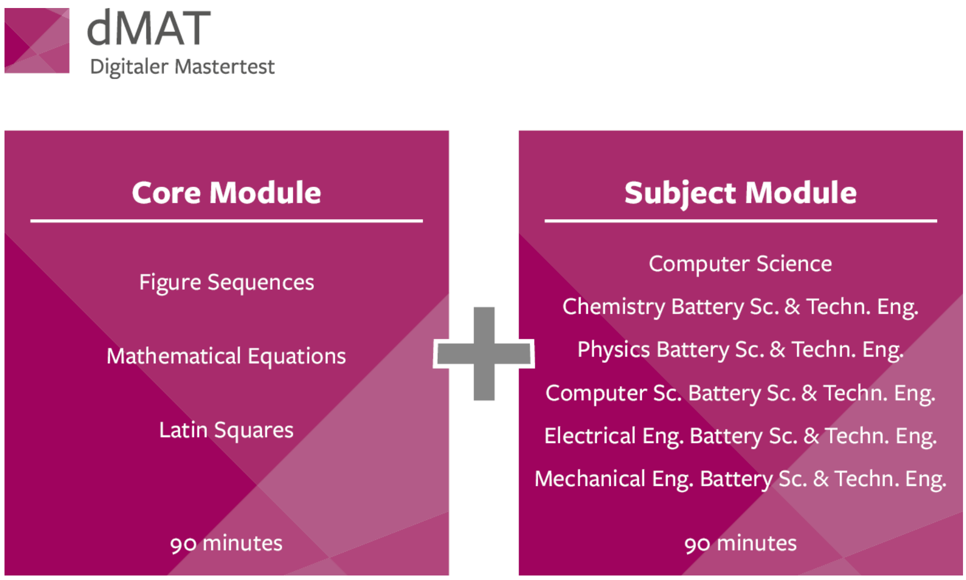
The digital Master Assessment Test (dMAT) is a new study aptitude test used for the admission of international applicants to Master's degree programmes in Germany. Taking the test allows a fair comparison of applicants across different national education systems.

The dMAT is offered digitally. Developed in close professional collaboration with German universities, the test accurately reflects the requirements of the intended degree programmes and is thus a valid instrument for assessing study ability for master's degree programmes.

The dMAT exams are evaluated centrally at the TestDaF Institute in Bochum. The test is standardized, which ensures that all participants can be compared with each other. In addition, the format of the test is based on the internationally recognized standards for psychodiagnostic tests, the test is reliable and objective.

Structure of the dMAT

The dMAT consists of two parts: A Core Module that tests general study aptitude, and subject-specific modules that test subject-specific aptitude as well as the ability to apply the knowledge acquired in the course of study. The following graphic illustrates the structure of the test and the current subject modules.



The duration of the exam itself is about three hours with a break of 30 minutes between the two parts of the exam.

The **Core Module** measuring general study ability consists of three subtests that measure general cognitive abilities relevant to a master's degree programme in Germany. To a certain extent, the core module allows participants to be compared across the respective subject modules.

The production of the **Subject Modules** is based on extensive scientific studies by experts, so that the exam content is representative of the respective fields of study. The examination tasks are knowledge-based and consist of a combination of a typical subject-related problem (input) and corresponding single-choice questions. The dMAT therefore requires subject knowledge and application skills, but not memorized factual knowledge. You can familiarize yourself with the exact requirements and instructions of the individual task types in the next sections.

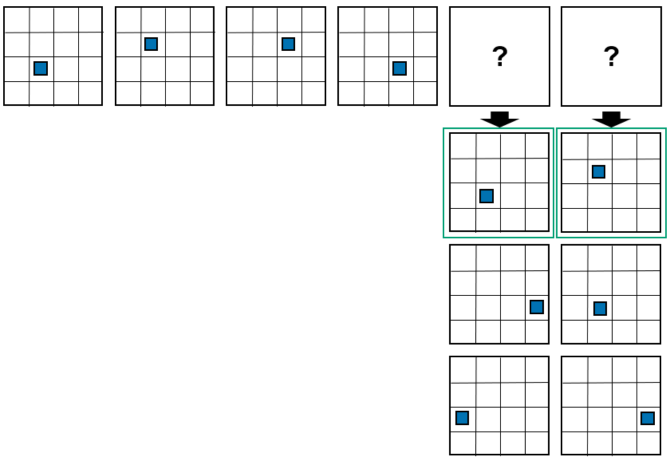
Please note: You may not take notes throughout the exam.

Core Module – Instructions and Exercises

Core Module	Figure Sequences
Instructions	

In this task you will see a series of pictures (matrices). The figures in the matrices can change their **position**, **colour**, and/or **orientation** from one matrix to the next according to specific rules. It is your task to continue the series logically and to determine what the next two matrices look like.

Example Task



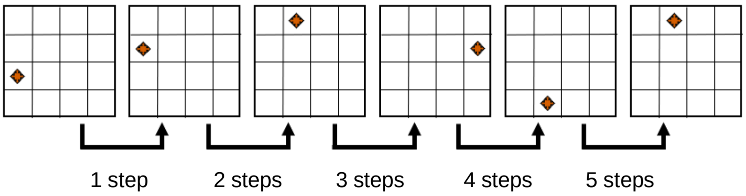
Solution

The blue square always moves one field clockwise within the four middle fields. Therefore, for the fifth matrix the first response option is correct, and for the sixth matrix as well.

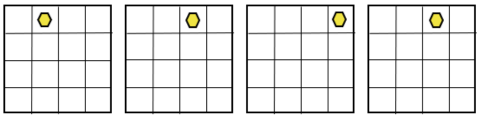
Rules

- Figures can change their colour.
- Figures can rotate around their own axis.
- Figures can move in the matrix. Vertical, horizontal, and diagonal movements are allowed. Figures cannot change from one diagonal movement to another type of movement.

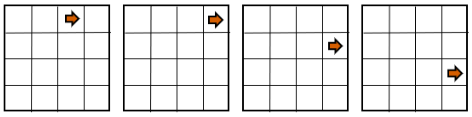
- Figures can also change their movement, colour or orientation by x + 1. Example: If a figure moves one step from matrix 1 to matrix 2, it moves 2 steps from matrix 2 to matrix 3, then 3 steps, etc.



- Figures cannot disappear or overlap.
- Figures cannot leave the matrix. If they come up against an outer boundary, they can EITHER
- bounce off OR



- move along the outer boundary.



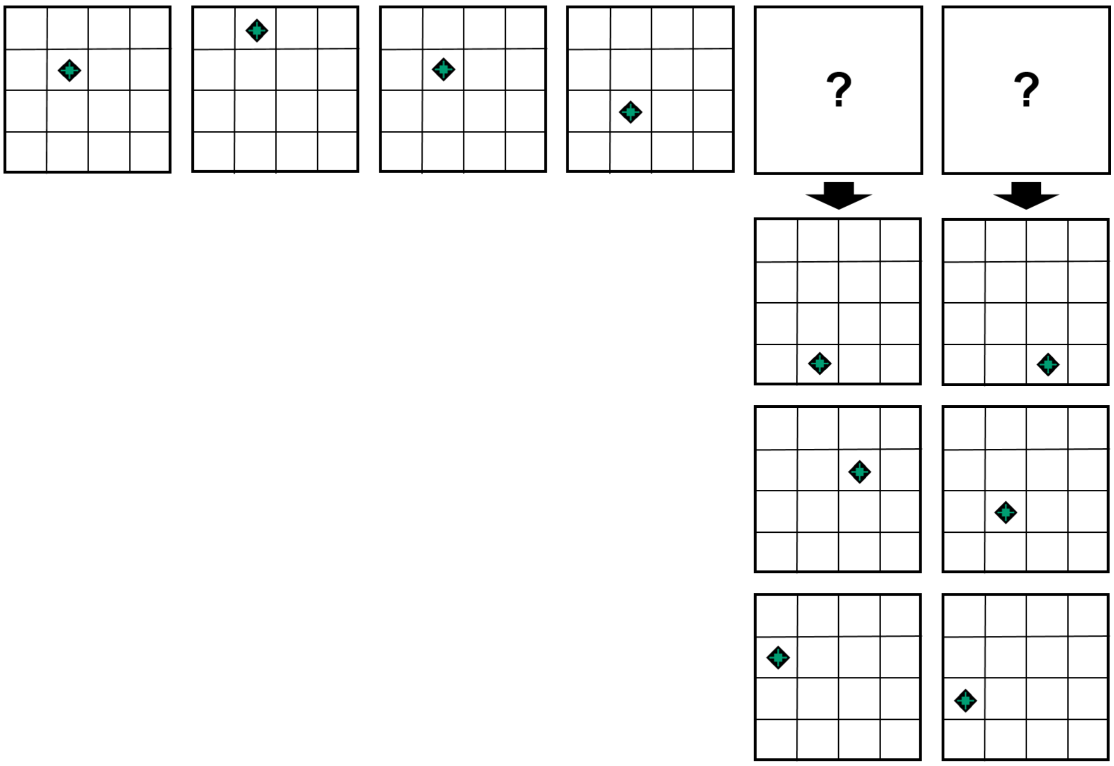
In the exam you have a total of **25 minutes** for **20** series of matrices. Please be as quick and accurate as possible. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

Core Module **Figure Sequences**

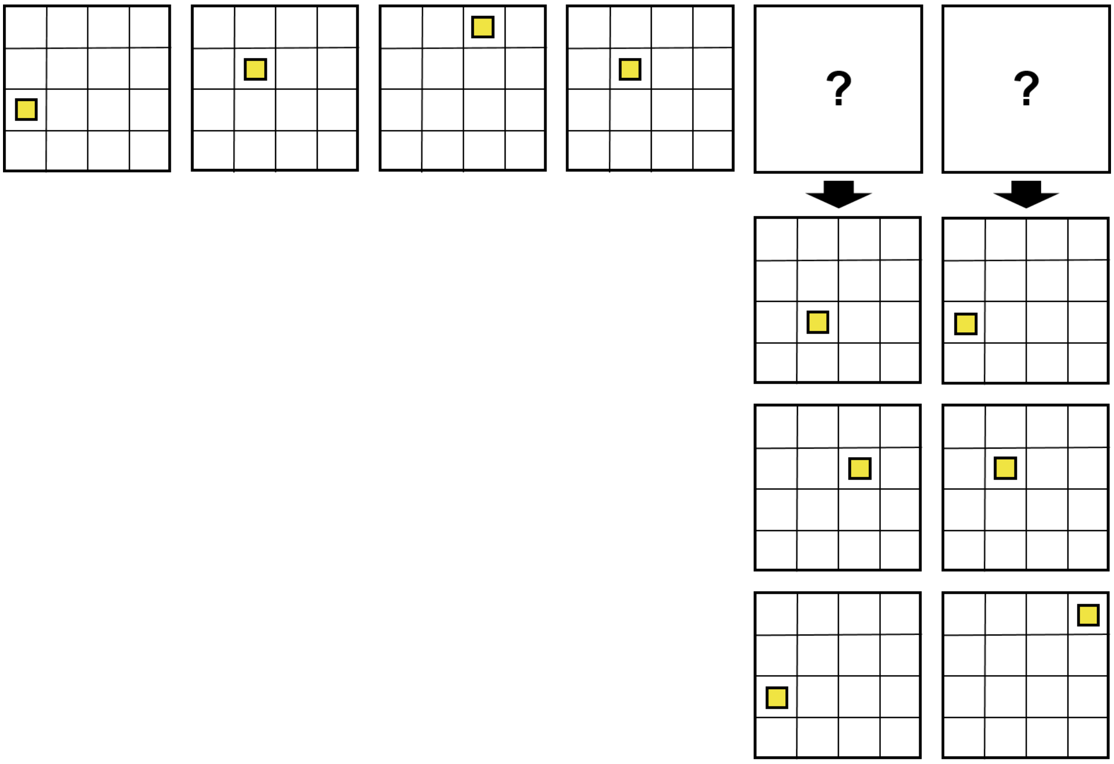
Exercises

For the task type **Figure Sequences** there are six exercises available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

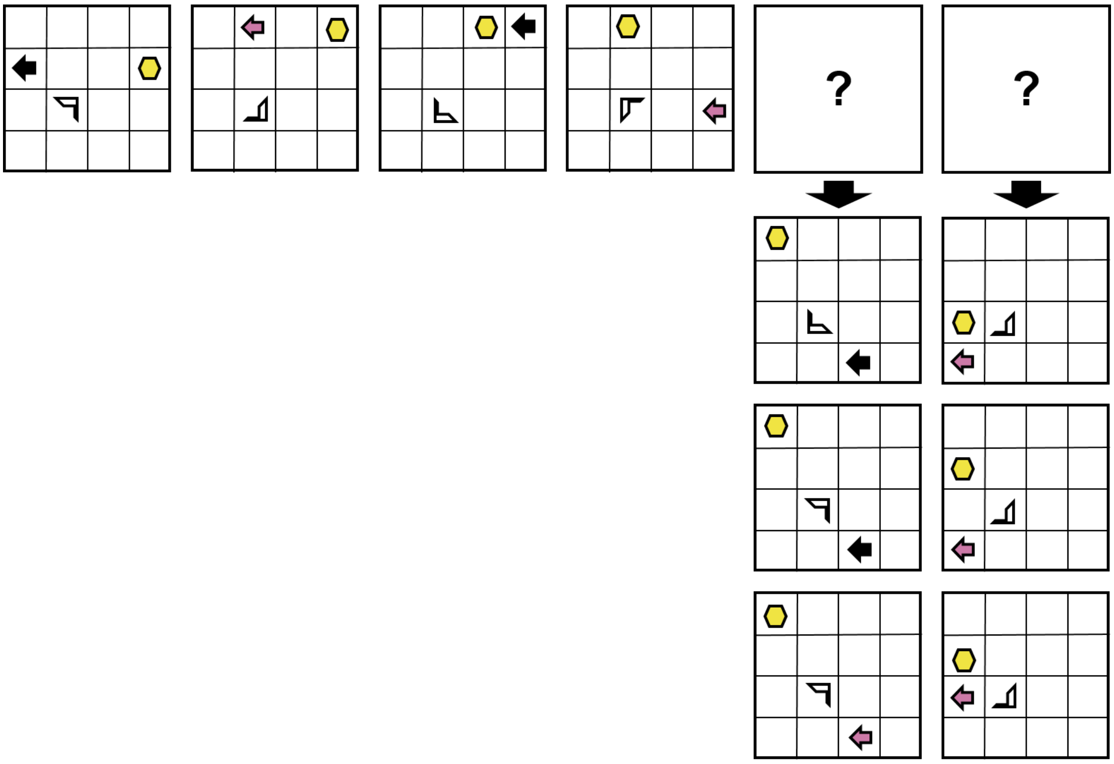
Exercise 1 – Difficulty: low



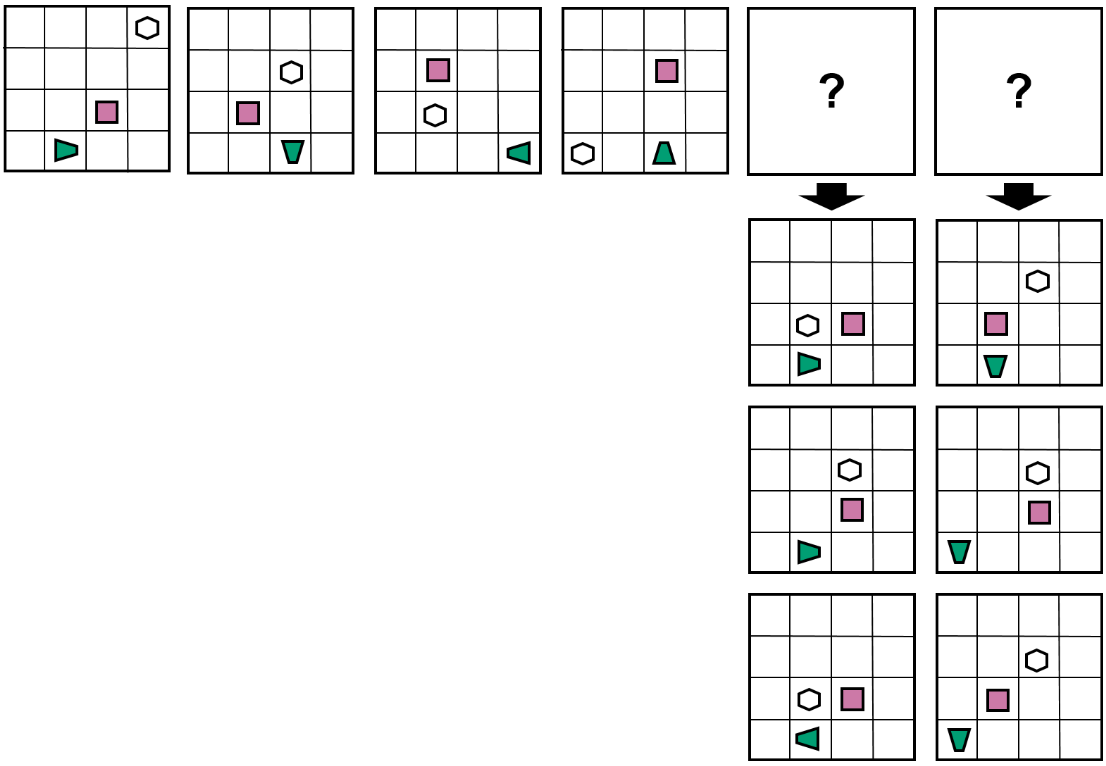
Exercise 2 – Difficulty: low



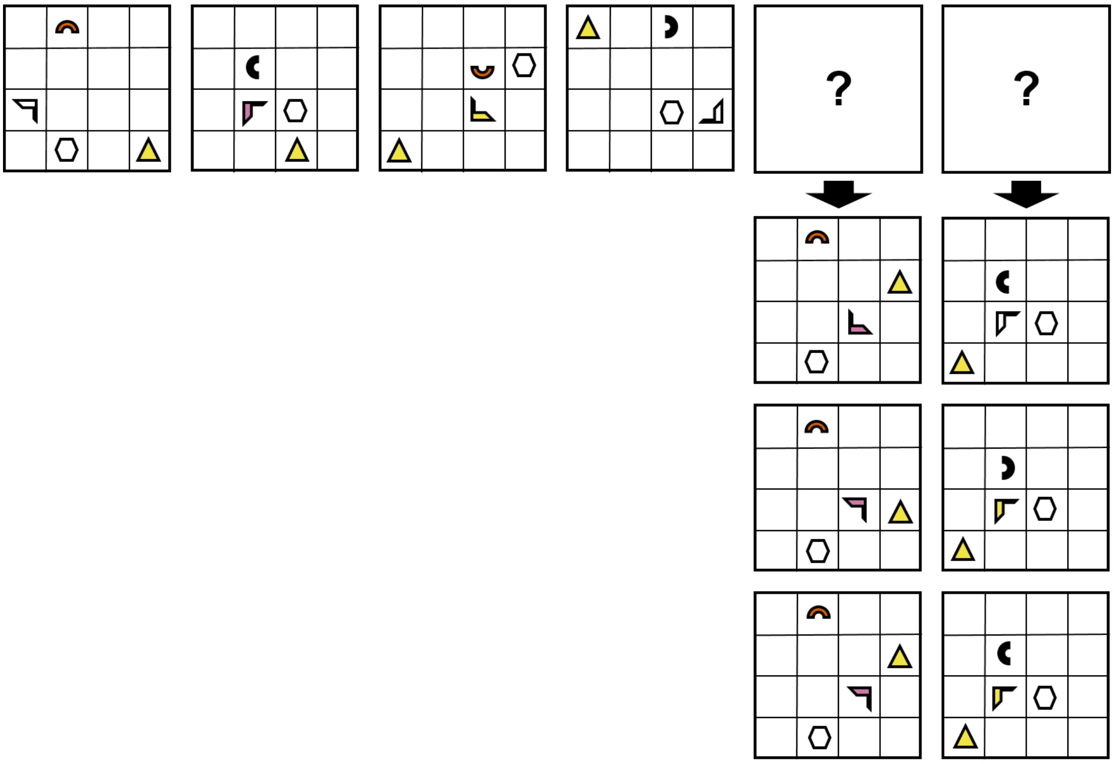
Exercise 3 – Difficulty: medium



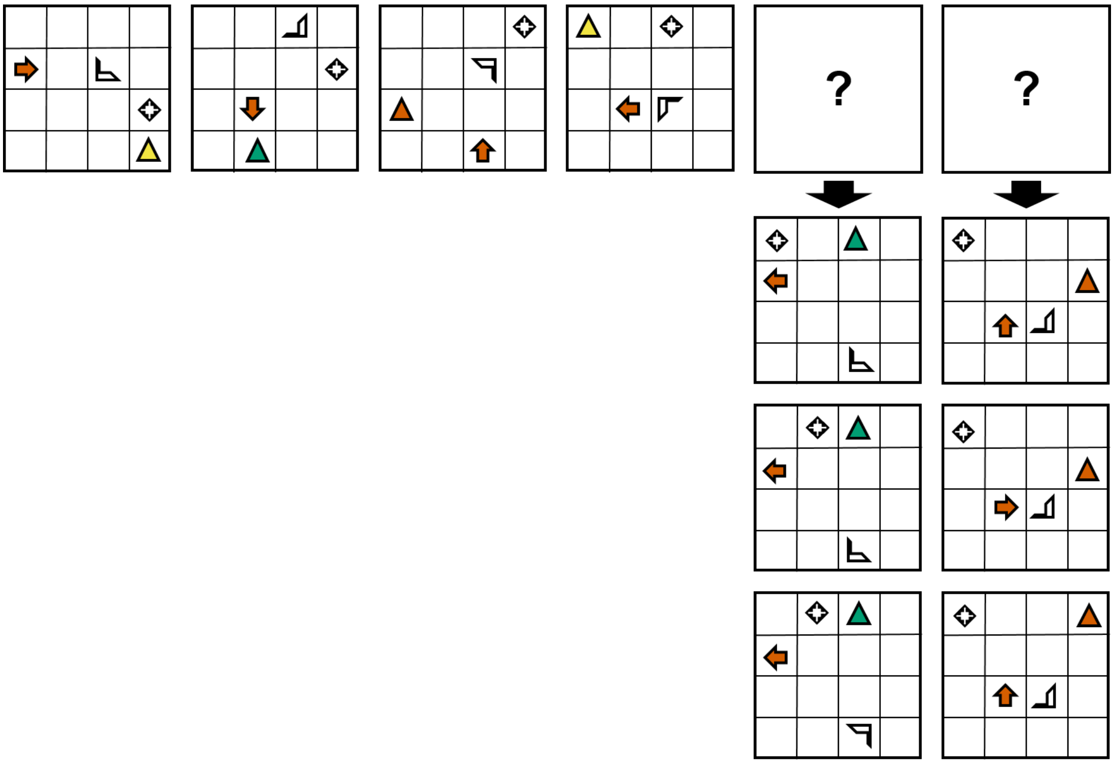
Exercise 4 – Difficulty: medium



Exercise 5 – Difficulty: high

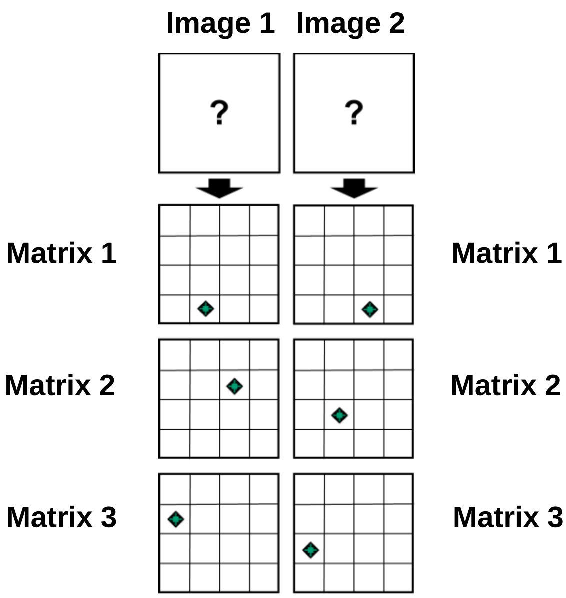


Exercise 6 – Difficulty: high



Core Module	Figure Sequences
Exercises – Solutions	

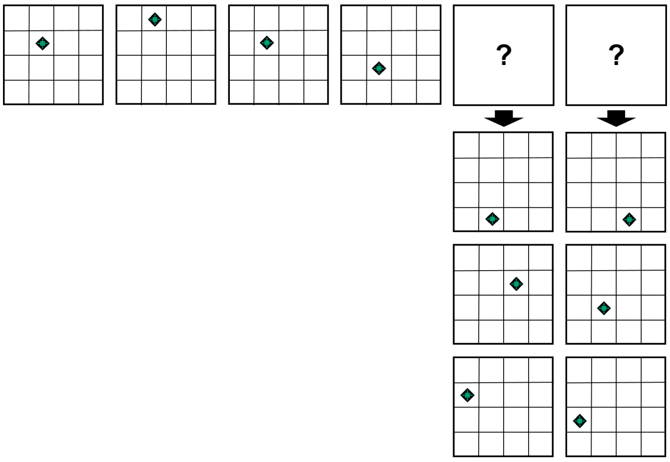
Note on the solution key



Solution – Exercise 1

Image 1: Matrix 1

Image 2: Matrix 2

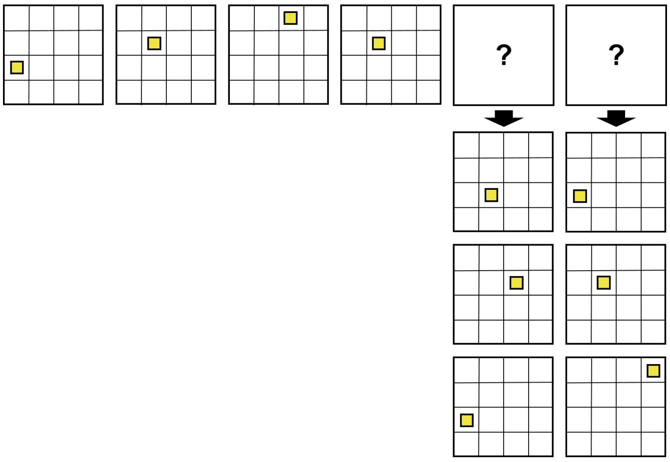


The symbol  moves vertically one field at a time in the second column and bounces off the upper or lower boundary.

Solution – Exercise 2

Image 1: Matrix 3

Image 2: Matrix 2

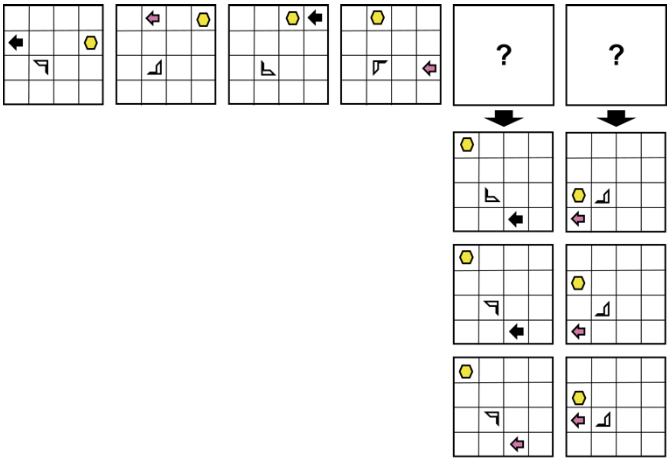


The symbol always moves one space diagonally upwards to the right from its starting position until it bounces off the upper boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 3

Image 1: Matrix 2

Image 2: Matrix 2



The symbol moves along the outer borders clockwise by two squares at a time. It changes its colour alternately from black to pink.

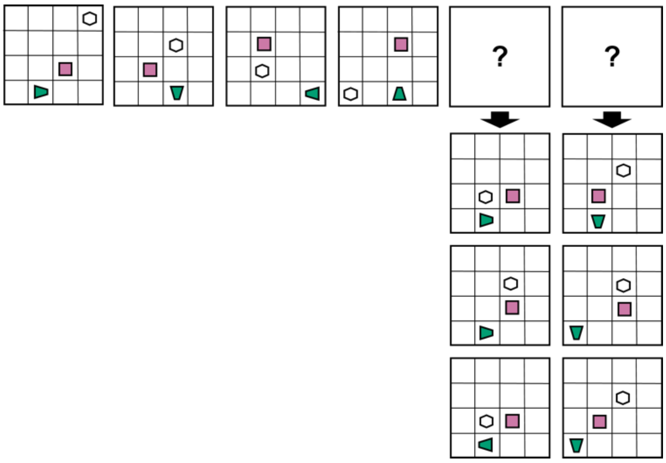
The symbol rotates 90 degrees to the right from image to image.

The symbol moves along the outer borders counter clockwise one space at a time.

Solution – Exercise 4

Image 1: Matrix 1

Image 2: Matrix 3



The symbol moves horizontally by one field in the fourth row and bounces off the right or left border. It rotates 90 degrees to the right from image to image.

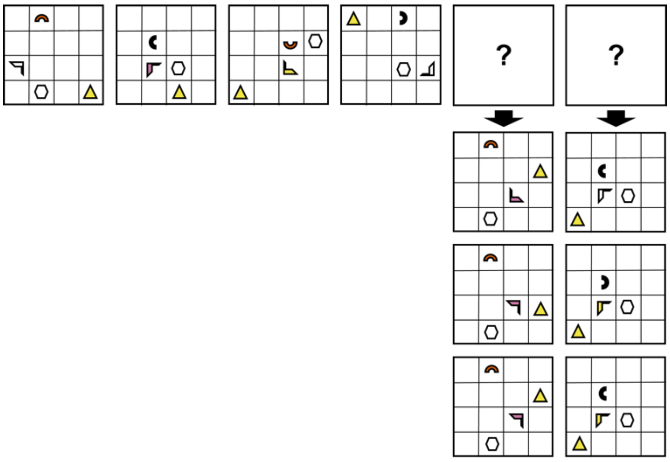
The symbol moves from its starting position one field at a time from image to image. The order of the directions in which the symbol moves is: left, up, right, down, and so on.

The symbol moves from its starting position diagonally downwards to the left until it bounces off the bottom left corner and returns the same way to the top right corner (diagonally upwards to the right).

Solution – Exercise 5

Image 1: Matrix 3

Image 2: Matrix 3



The symbol moves along the outer borders clockwise by x + 1 fields (i.e. from matrix 1 to matrix 2 one field, from matrix 2 to matrix 3 two fields, and so on).

The symbol moves horizontally by one field in the third row and bounces off the right or left border. In doing so, it turns 90 degrees to the left from image to image and changes its colour from white to pink to yellow , and so on.

The symbol moves from its starting position one field at a time from image to image. The sequence of directions in which the symbol moves is: down, right, up, left, and so on. It turns 90 degrees to the left and changes its colour alternately from orange to black.

The symbol moves diagonally upwards to the right from its starting position until it bounces off the right boundary and returns to the starting position in the same way (diagonally downwards to the left). Once there, it bounces off the lower boundary and moves diagonally upwards to the right again.

Solution – Exercise 6

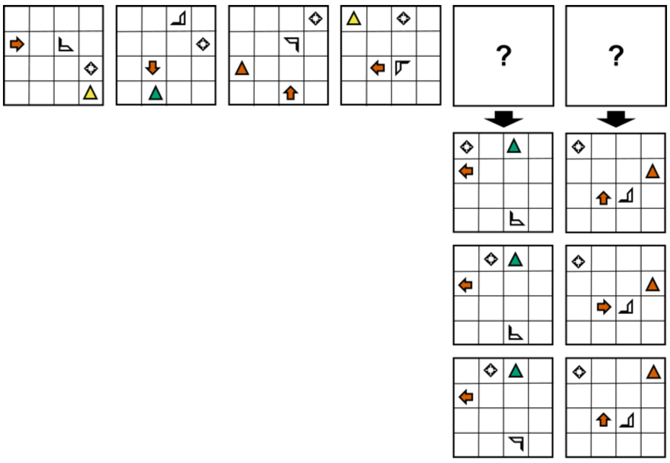


Image 1: Matrix 2

Image 2: Matrix 1

The symbol moves from its starting position diagonally downwards to the right until it bounces off the lower boundary and returns to the starting position in the same way (diagonally upwards to the left). Once there, it bounces off the left boundary and moves diagonally down to the right again. The symbol always turns x + 1 times to the right by 90 degrees. I.e. from matrix 1 to matrix 2 it rotates once by 90 degrees to the right. From matrix 2 to matrix 3 it rotates twice by 90 degrees to the right, and so on.

The symbol moves vertically by one field in the third column and bounces off the upper or lower border. It rotates 90 degrees to the left from image to image.

The symbol moves along the outer borders counter clockwise one space at a time.

The symbol moves along the outer borders clockwise by two spaces at a time. In the process, it changes colour from yellow to green to orange , etc.

Core Module	Mathematical Equations
Instructions	

In this task, you are supposed to solve systems of equations in such a way that all requirements are met. One system of equations always consists of several single equations.

Your task is to find out which numbers must be used for the unknowns (letters) in the equations so that all equations are correct.

There is always only one solution for each letter, in which all requirements are met.

Each letter can be an integer between 1 and 20.

Example 1

A + 2 = **B**

B = 6

What number does **A** correspond to so that the equations are solved correctly?

Solution of Example 1

Because of the second equation, you know that **B** = 6. Replace **B** with the number 6 in the first equation and you get **A** + 2 = 6. Solve the first equation and you get **A** = 6 – 2 = 4. Therefore, the solution of the first example is **A** = 4. Any other solution is wrong.

Example 2

B = 2 × **A**

B + **A** = 12

What numbers do **A** and **B** correspond to so that the equations are solved correctly?

Solution of Example 2

The first equation defines that **B** = 2 × **A**. Putting this information in the second equation gives 2 × **A** + **A** = 12 or 3 × **A** = 12. Rearranging this equation gives **A** = 12 ÷ 3 = 4. Putting the number 4 into the first or second equation for **A** gives **B** = 8. Therefore, the solution of the second example is **A** = 4 and **B** = 8. Any other solution is wrong.

In the exam you have **25 minutes** to solve **20 systems of equations**. Please be as quick and accurate as possible. You are not allowed to take notes in the exam.

Core Module	**Mathematical Equations**
Exercises	

For the task type **Mathematical Equations**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

$$7 + A = 14$$

$$B - 3 = A$$

Exercise 2 – Difficulty: low

$$B \div 2 = A$$

$$B - A = 8$$

Exercise 3 – Difficulty: medium

$$3 \times C = A$$

$$A + C = 8$$

$$2 \times A + 2 \times C = B$$

Exercise 4 – Difficulty: medium

$$18 - B = A$$

$$3 \times A = C$$

$$B \div 2 = A$$

Exercise 5 – Difficulty: high

$$A - B + C - D = 2$$

$$10 \times B = C$$

$$5 \times B = A$$

$$11 + B = D$$

Exercise 6 – Difficulty: high

$$C + D - A = 1$$

$$5 \times C = D$$

$$13 - C = A$$

$$3 \times C - 1 = B$$

Core Module	Mathematical Equations
Exercises – Solutions	

Solution – Exercise 1

$7 + A = 14$
$B - 3 = A$

$A = 7$
$B = 10$

The first equation makes it clear that $A = 7$ if you subtract 7 on both sides. If you insert this information into the second equation, you get $B - 3 = 7$. If you add 3 on both sides, you get the solution $B = 10$.

Solution – Exercise 2

$B \div 2 = A$
$B - A = 8$

$A = 8$
$B = 16$

Multiplying by 2 on both sides in the first equation gives $B = 2A$. Replacing the variable B in the second equation with this information gives $2A - A = 8$. This means $A = 8$. Substituting the solution for A in the first equation gives $B \div 2 = 8$. Multiplying both sides by 2 gives $B = 16$.

Solution – Exercise 3

$3 \times C = A$
$A + C = 8$
$2 \times A + 2 \times C = B$

$A = 6$
$B = 16$
$C = 2$

With the information from the first equation ($3 \times C = A$ or $A = 3C$), A can be replaced in the second equation so that it can be solved for C: $3C + C = 8$ or $4C = 8$. If you divide by 4 on both sides, you get $C = 2$. Thus, the solution of A can be calculated by substituting the value of C into the first equation: $3 \times 2 = A$. Therefore, $A = 6$. By substituting the solutions for A and C, the third equation can be solved for B: $2 \times 6 + 2 \times 2 = B$. Therefore, $B = 16$.

Solution – Exercise 4

$18 - B = A$
$3 \times A = C$
$B \div 2 = A$

$A = 6$
$B = 12$
$C = 18$

If you multiply by 2 on both sides in the third equation, you get $B = 2A$ (alternatively, you can also continue the calculation with $0.5B = A$, for example). If you replace B with this information in the first equation, you get $18 - 2A = A$. If you add 2A on both sides, you get $18 = 3A$. If you now divide by 3, you get $A = 6$. This information can be inserted into the third equation, so that you get $B \div 2 = 6$. If you multiply by 2 on both sides, you get $B = 12$. If you insert the result for A into the second equation, you get $3 \times 6 = C$, so $C = 18$.

Solution – Exercise 5

$A - B + C - D = 2$
$10 \times B = C$
$5 \times B = A$
$11 + B = D$

$A = 5$
$B = 1$
$C = 10$
$D = 12$

The information given in equations two, three and four for the variables A, B and C can be inserted into the first equation so that it can be solved for B: $5B - B + 10B - (11 + B) = 2$. If you dissolve the bracket, you get $5B - B + 10 B - 11 - B = 2$ or $13B - 11 = 2$. If you add 11 on both sides, you get $13B = 13$. If you divide by 13, you get the solution $B = 1$. This information can be inserted into the other equations and solved for the respective missing variable: $10 \times 1 = C$ or $C = 10$, $5 \times 1 = A$ or $A = 5$ and $11 + 1 = D$ or $D = 12$.

Solution – Exercise 6

$C + D - A = 1$
$5 \times C = D$
$13 - C = A$
$3 \times C - 1 = B$

$A = 11$
$B = 5$
$C = 2$
$D = 10$

The information given in equations two and three for the variables A and D can be inserted into the first equation, so that it can be solved for C: $C + 5C - (13 - C) = 1$. Dissolving the bracket gives $C + 5C - 13 + C = 1$ or $7C - 13 = 1$. Adding 13 on both sides gives $7C = 14$. Dividing by 7 gives the solution $C = 2$. This information can be inserted into the other equations and solved for the respective missing variable: $5 \times 2 = D$ or $D = 10$, $13 - 2 = A$ or $A = 11$ and $3 \times 2 - 1 = B$ or $B = 5$.

Core Module	Latin Squares
Instructions	

In this task you will see a 5x5 grid (a square containing 5 rows and 5 columns).

Some fields of the grid contain letters. Each letter can only appear once in each row and each column. Only the letters that are shown as response options (the row next to the grid) can appear in the grid.

Your task is to decide which letter belongs in the field with the question mark. Sometimes you need to fill in other fields in your mind before you can figure out what letter should replace the question mark.

If you know what the correct solution for the question mark field is, click on the correct response in the solution row.

	?			
	A			
	E			
	D			
C	B			

A
B
C
D
E

Next, you will see two examples.

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Example 1

			D	
			A	
			C	
C	D	A	?	
			E	D

A
B
C
D
E

			D	
			A	
			C	
C	D	A	B	
			E	D

A
B
C
D
E

Solution of Example 1

In the first example, “B” needs to replace the red question mark, because all other letters D, A, C, and E already appear in this column.

Example 2

A	E	C	D	
				A
				E
				?
	B			C

A
B
C
D
E

A	E	C	D	B
				A
				E
				D
	B			C

A
B
C
D
E

Solution of Example 2

In the second example, you first need to fill in “B“ in the first row of the last column. “B” is the only letter, which does not already appear in this row and column. Then you can replace the question mark with “D”, because it is the only letter that does not appear in the last column.

In the exam you have 25 minutes for 20 tasks. Please be as quick and accurate as possible! If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

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Core Module	Latin Squares
Exercises – Solutions	

For the task type **Latin Squares**, six exercises are available, two each in the difficulty levels low, medium and high. On the following pages you can see the solutions including the solution paths. Practice with these exercises without taking notes, as you will not have any helping tools available to you in the exam either.

Exercise 1 – Difficulty: low

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Exercise 2 – Difficulty: low

		?		
			D	A
		E		D
A	D			B
D	B		C	

Exercise 3 – Difficulty: medium

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Exercise 4 – Difficulty: medium

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

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Exercise 5 – Difficulty: high

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Exercise 6 – Difficulty: high

?				C
	D	E	B	A
B		D	A	
	B	C		D

Core Module	Latin Squares
Exercises – Solutions	

Note on the solution key

	α	β	γ	δ	ε
1	B	?	A	D	
2	A	B	E	C	
3		A			
4	C				
5	D	E		B	

Solution – Exercise 1

Solution = C

B	?	A	D	
A	B	E	C	
	A			
C				
D	E		B	

Solution path:

- In column β, C and D are missing.
- C is already in row 4, so D must be inserted in β4.
- Consequently, C must be inserted in the place of the question mark.

Solution – Exercise 2

Solution = D

		?		
			D	A
		E		D
A	D			B
D	B		C	

Solution path:

- In the place of the question mark, D must be inserted because D is already given in all other columns and rows.

Solution – Exercise 3

Solution = B

A			B	
	B	A		
	E	D		
E	C		A	D
		E		?

Solution path:

- In column γ, B and C are missing. At position γ4, only B can be inserted, since there is already a B in row 1. This also applies in γ4, or line 4 vice versa for C. Consequently, only a C can be inserted in γ1.
- A and D are missing in column β. A can only be in position β5, because A is already present in row 1. Consequently, only a D can be in position β1.
- From this follows that only one E can be inserted in ε1.
- In row 3 it is now noticeable that A can only be in position ε3, as it is already present in all columns and rows.

- Since a B still has to be inserted in column ε, and there is already a B in line 2, it can only be inserted at the position of the question mark.

Solution – Exercise 4

Solution = D

	E		C	B
?			A	
		A	E	D
B	A		D	
	D	C		

Solution path:

- A and D are missing in the first row. A can only be inserted at position α1, since it is already in column γ. Consequently, D must be in position γ1.
- It is now noticeable that D is already present in four different rows and columns and can thus only be used in the place of the question mark.

Solution – Exercise 5

Solution = D

			C	
	C	?	E	
	E		B	C
A	B		D	E
	D	E	A	

Solution path:

- Only C can be inserted at position γ4.
- In row 3, A and D are missing. At position γ3, only an A can be inserted because it is already present in column α. Consequently, only a D can be inserted at position α1.
- Only A can be inserted at position β1.
- Only E can be inserted at position α1, as it is already present in all other rows and columns.
- Furthermore, C and B are missing in row 5, whereby only a C is inserted at position α5, since it is already present in column ε. Consequently, there is a B at position ε5.
- In row 1, D and B are still missing. Since B is already in column ε, B must be inserted at position γ1 and D at position ε1.
- At the position of the question mark, D must be inserted, since all other letters are already present in column γ.

Solution – Exercise 6

Solution = E

?				C
	D	E	B	A
B		D	A	
	B	C		D

Solution path:

- At position α3, only a C can be inserted, since all other letters are already in line 3.
- In row 5, A and E are missing. A must be in position α5 because it is already in column δ. Consequently, E is in position δ5.
- In row 4, C and E are missing. Only a C can be inserted at position β4, as it is already present in column ε. Consequently, there is an E at position ε4.
- At position ε2, only a B can be inserted, as all other letters in column ε are already present.
- In column γ, A and B are missing. At position γ1, only a B can be inserted, since there is already a B in the second row. Consequently, A must be inserted in γ2.
- In the first row, A, D and E must be inserted. A must be inserted in β1 because it is already present in all the other columns. Since E is already present in column δ, it must therefore be inserted in the position of the question mark.

Subject Module – Instructions and Exercises

Subject Module	General Academic Module
General Instructions	

In this task type you see a text and a number of questions which you have to answer. There are 4 answer options for each question.

For each question, there is only one correct solution.

The text, the questions and the answer options may contain figures, tables and formulas.

For working on the entire subject test in the exam, you have 90 minutes in total. If you do not know an answer, please guess which answer might be correct. You are not allowed to take notes in the exam.

For practice and illustration of the subject module tasks, four exercises are available here.

Please note that the sample questions do not necessarily reflect the difficulty level of the questions on the actual test. Furthermore, the questions represent only a selection of the possible topics that may be covered.

The possible topics of the tasks in the actual exam cover a variety of areas including mathematics, computational sciences, natural sciences, engineering, business administration, economics, social sciences and humanities.

Subject Module | General Academic Module

Exercise 1

Vector Calculations

While so-called scalar values consist of a single number (for example a = 2), vectors are formed from several numbers. Vectors can define a point in an n-dimensional vector space using n numbers (components). This document will focus on vectors in two- and three-dimensional vector spaces, hence consisting of up to three components (see Figure 1 as an example for the two-dimensional case).

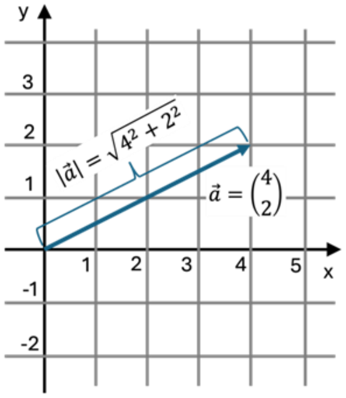


Figure 1. Example of a two-dimensional vector (including calculation of the vector's length)

When adding two vectors, the individual components are added. This also applies to the subtraction of vectors. When a vector is multiplied by a scalar, all its components are multiplied by this scalar. If the scalar is greater than 1, the length of the vector is accordingly extended. If the scalar is greater than 0 but less than 1, the length of the vector is shortened. For vector representations in the Cartesian coordinate system, the length of a vector (magnitude of the vector) can be calculated using the Pythagorean theorem or its generalisation to several components (see Figure 1 as an example for the two-dimensional case).

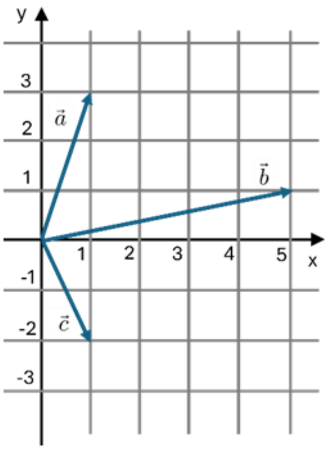
Among the most important operations in vector calculations are the *scalar product* and the *vector product*, also known as the cross product. Additionally, a so-called *triple product* has been defined, which represents a combination of scalar and vector products. The following table provides an overview, where the angle φ describes the angle between the two vectors.

Name	Notation	Calculation Rule	Further Relationships
Scalar product	$\vec{a} \cdot \vec{b}$	$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \cdot \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z$	$\vec{a} \cdot \vec{b} = \lvert\vec{a}\rvert \, \lvert\vec{b}\rvert \cos(\varphi)$
Vector product	$\vec{a} \times \vec{b}$	$\vec{a} \times \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$	$\lvert\vec{a} \times \vec{b}\rvert = \lvert\vec{a}\rvert \, \lvert\vec{b}\rvert \sin(\varphi)\ with\ 0 \leq \varphi \leq \pi$
Triple product	$[\vec{a}\ \vec{b}\ \vec{c}]$	$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$	

Table 1. Overview of vector calculations for vectors in a three-dimensional Cartesian coordinate system

Question 1

Which vector results when the operation $\vec{s} = \vec{a} + \vec{b} - \vec{c}$ is performed?



a) $\vec{s} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$
b) $\vec{s} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$
c) $\vec{s} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$
d) $\vec{s} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$

Question 2

What statement is true when a vector is multiplied by the number "-2"?

a) The length of the vector doubles. The direction of the vector remains the same.
b) The length of the vector halves. The direction of the vector is reversed.
c) The length of the vector doubles. The direction of the vector is reversed.
d) The length of the vector halves. The direction of the vector remains the same.

Question 3

The type of result from the three operations presented in the table is not always the same. Which statement is correct?

a) Forming a scalar product results in a scalar; forming a vector product results in a vector; forming a triple product results in a vector.
b) Forming a scalar product results in a scalar; forming a vector product results in a vector; forming a triple product results in a scalar.
c) Forming a scalar product results in a vector; forming a vector product results in a vector; forming a triple product results in a scalar.
d) Forming a scalar product results in a scalar; forming a vector product results in a scalar; forming a triple product results in a scalar.

Question 4

For the following vectors, the scalar product was calculated. Which result of the calculation is correct?

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

a) 21
b) 32
c) $\sqrt{14} + \sqrt{77}$
d) 126

Question 5

With the scalar product, it is possible to determine the angle between two vectors. What angle φ is formed between the following two vectors? $\vec{a} = \binom{1}{2}$, $\vec{b} = \binom{2}{1}$ (arccosine is the inverse function of the cosine)

a) $\varphi = \arccos\left(\frac{4}{5}\right)$
b) $\varphi = \arccos\left(\frac{4}{\sqrt{5}}\right)$
c) $\varphi = \arccos\left(\frac{5}{4}\right)$
d) $\varphi = \arccos\left(\frac{4}{\sqrt{3}}\right)$

Question 6

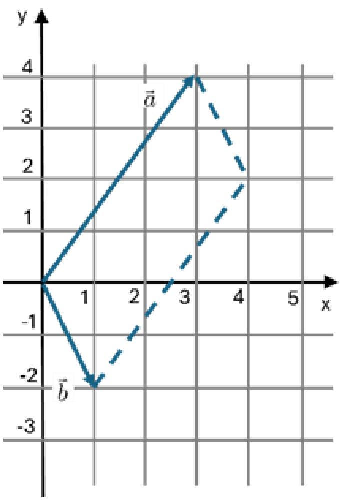
Which vector results when the vector product of the following vectors is determined?

$$\vec{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$$

a) $\vec{a} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$
b) $\vec{a} = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$
c) $\vec{a} = \begin{pmatrix} 9 \\ 7 \\ 5 \end{pmatrix}$
d) $\vec{a} = \begin{pmatrix} 18 \\ 10 \\ 4 \end{pmatrix}$

Question 7

The magnitude of a result vector (vector product) corresponds to the area of the parallelogram spanned. How large is the following area? (Note: The formula for the three-dimensional case can also be used here if the z-coordinates are set to zero.)



a) $\sqrt{5}$ square units

b) 5 square units

c) $\sqrt{10}$ square units

d) 10 square units

Question 8

The triple product can be used to calculate a volume. The volume pertains to the space encompassed by six parallelograms defined by the three vectors. Which statement is correct for <u>all</u> possible cases when the triple product has a value of "0"?

a) The vectors lie in a plane.

b) The vectors are pairwise perpendicular.

c) The vectors are linearly independent.

d) The vectors sum up to the zero vector.

Subject Module	General Academic Module
Exercise 1 – Solutions	

Question 1

Solution: A

The vectors are added or subtracted component by component. From the diagram, $\mathrm{a} = (1{,}3)$, $\mathrm{b} = (5{,}1)$, and $\mathrm{c} = (1, -2)$, so $\mathrm{a} + \mathrm{b} - \mathbf{c} = \big(1 + 5 - 1,\ 3 + 1 - (-2)\big) = (5{,}6)$

Question 2

Solution: C

Multiplication by -2 doubles the magnitude of the vector because the absolute value of the scalar is 2. The negative sign reverses the direction of the vector.

Question 3

Solution: B

A scalar product results in a scalar, whereas a vector product results in a vector. The triple product combines both operations and finally results in a scalar.

Question 4

Solution: B

The scalar product is calculated by multiplying corresponding components and adding the products: $\mathrm{a} \cdot \mathrm{b} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 32$

Question 5

Solution: A

The scalar product is $\mathrm{a} \cdot \mathrm{b} = 1 \cdot 2 + 2 \cdot 1 = 4$. Since both vectors have the magnitude $\sqrt{5}$, $\cos(\varphi) = \frac{4}{\sqrt{5} \cdot \sqrt{5}} = \frac{4}{5}$, so $\varphi = \arccos\left(\frac{4}{5}\right)$.

Question 6

Solution: B

Using the cross product formula for $\vec{a} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$ gives $\mathrm{a} \times \mathrm{b} = (2 \cdot 4 - 1 \cdot 5,\ 1 \cdot 6 - 3 \cdot 4,\ 3 \cdot 5 - 2 \cdot 6) = \begin{pmatrix} 3 \\ -6 \\ 3 \end{pmatrix}$.

Question 7

Solution: D

The area of the parallelogram corresponds to the magnitude of the vector product. For the two-dimensional vectors shown, the determinant has the magnitude $|3 \cdot (-2) - 4 \cdot 1| = 10$.

Question 8

Solution: A

A triple product of 0 means that the volume of the solid spanned by the three vectors is 0. This is the case when the vectors are coplanar, i.e. when they lie in one plane.

Subject Module	General Academic Module
Exercise 2	

Hydrostatics

In incompressible fluids, pressure increases linearly with depth. A practical example of this is water pressure. To understand this, consider any point in the water. The column of water above this point exerts a weight force dependent on the height of the column. Since the fluid (here: water) is incompressible, there is a force at that point counteracting the weight force of the water column. Since there are infinitely many such hypothetical points in the water, the pressure cannot be redirected into other directions and thus acts equally in every direction. This principle is illustrated in Figure 1.

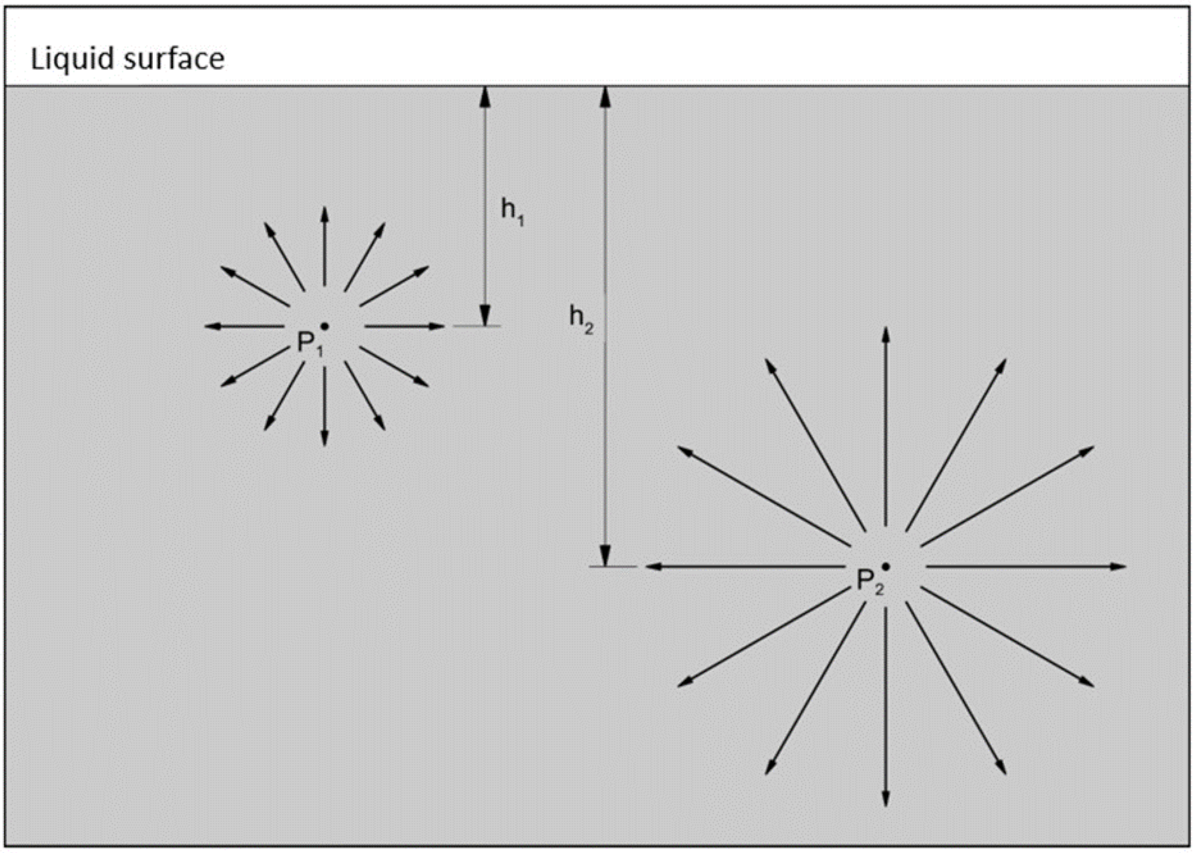


Figure 1. Two arbitrary points in a liquid: The deeper the point is below the surface, the greater the pressure.

The pressure at any point thus depends only on its vertical distance from the surface and on the density of the fluid.

An externally applied pressure adds to the pressure within the fluid. An example of this is the atmospheric pressure acting on a water surface. The normal atmospheric pressure at sea level is about 102,325 hPa, which is approximately 1 bar.

The total pressure in a fluid can be calculated as follows:

$$p(h) = \rho \cdot g \cdot h + p_0$$

(ρ = density of the medium, g = acceleration due to gravity, h = depth of the point below the liquid surface, p_0 = pressure at the surface, for example atmospheric pressure).

Roughly, the water pressure at a given depth can be estimated by the following simplifications:

$\rho_{water} = 1000\ \frac{kg}{m^3}$ and $g\ \approx 10\ N$

This results in the water pressure increasing by about 10^5 Pa, which is 1 bar, every 10 m.

The experience that bodies swim or float in liquids can be explained by pressure differences within the liquid. For a body swimming on the surface, a balance between weight force and pressure force is established. For a body floating in the fluid, a balance between the downward acting pressure forces and the upward acting pressure forces is established. The mass of the displaced fluid equals the mass of the body. The weight force acts at the body's centre of gravity (G), while the buoyant force acts at the centre of gravity of the part that is submerged. In any case, the sum of the pressure forces acting as buoyant force F_B is equal to the weight force F_G, so that:

$$F_G - F_B = 0$$

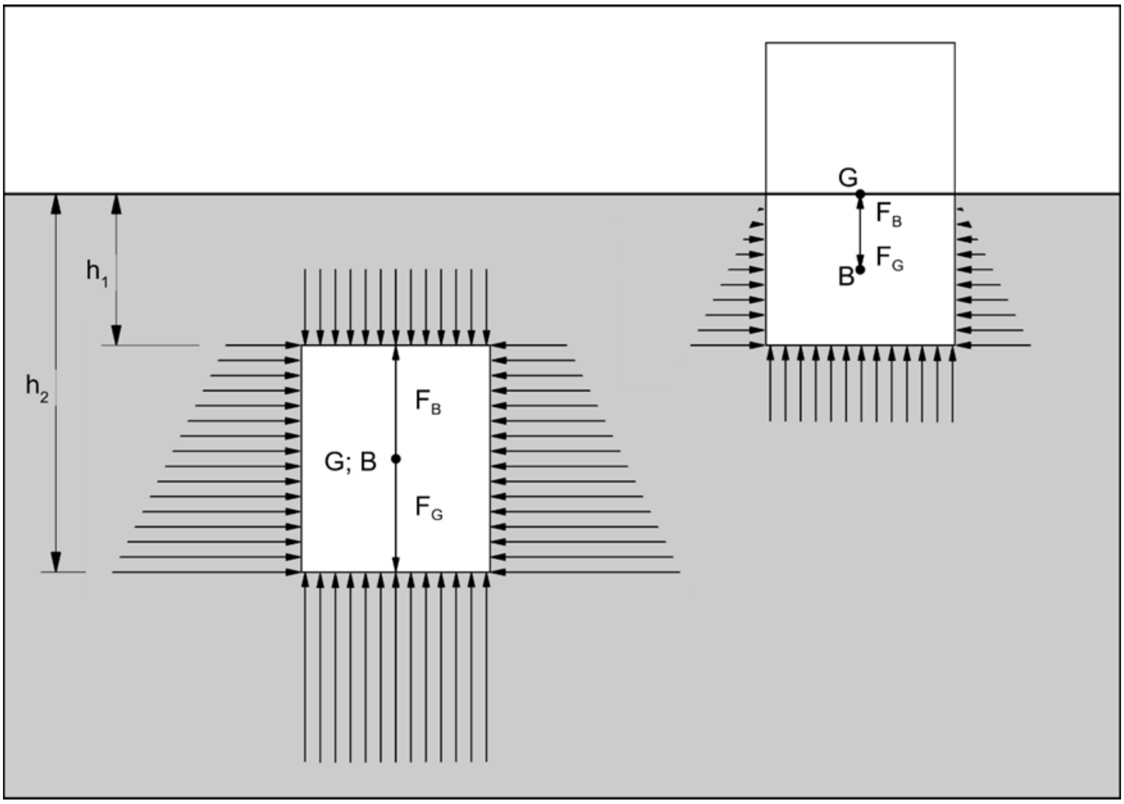


Figure 2. Two bodies in a liquid. Body 1 is fully submerged and floats. Body 2 is partially submerged and swims on the surface. In both cases, buoyant force and weight force are in balance.

Question 1

A submarine dives to a depth of 10,000 m below sea level. What is the approximate pressure on the hull of the submarine?

a) 1 bar

b) 10 bar

c) 100 bar

d) 1000 bar

Question 2

A body with a volume of 2 m³ floats in water. The top edge of the body is 3 m below the water surface, and the bottom edge of the body is 4 m below the water surface. What is the mass of the body?

a) 1,000 kg

b) 2,000 kg

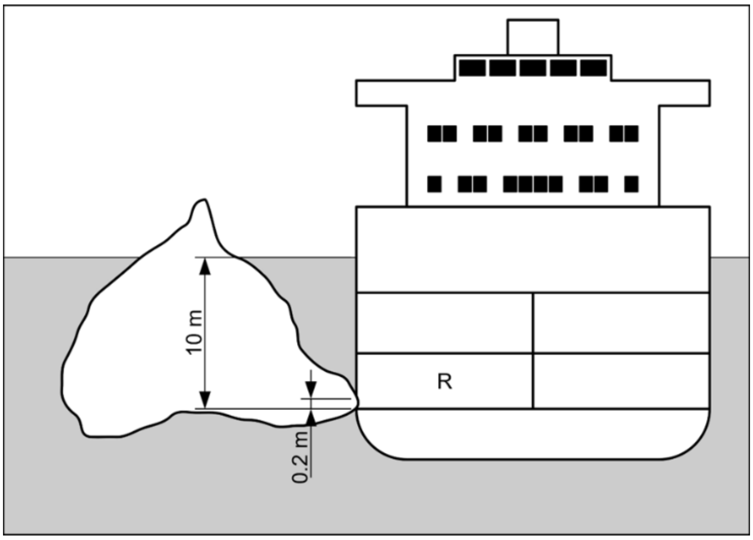
c) 3,000 kg

d) 4,000 kg

Question 3

A ship collides with an iceberg, tearing open the hull 10 m below the waterline. Normally, all spaces in the ship are connected to the outside air through a ventilation system. Just before the collision, the captain had all watertight doors closed, so now neither water nor air can escape from damaged rooms. Only the rectangular space R with a height of 24 m is affected. The tear is right above the floor and is 0.2 m high.

What will happen?



a) Room R will be completely flooded.

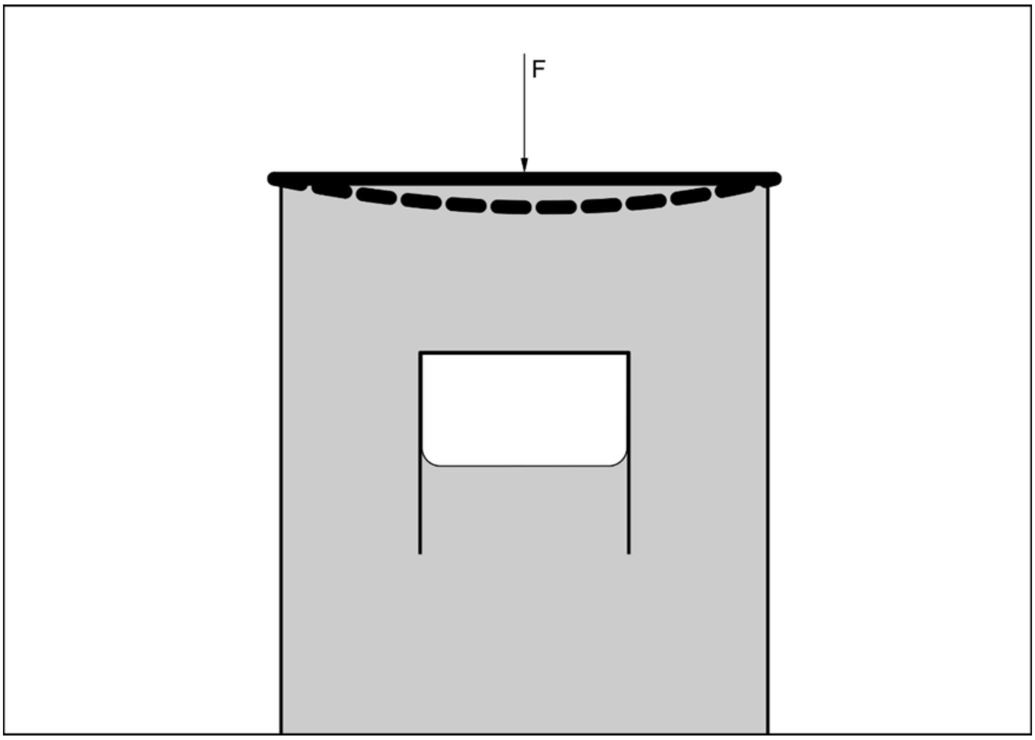
b) Water will enter room R and rise to about 12 m above the floor.

c) Water will enter room R and rise to about 0.2 m above the floor.

d) Nothing, because no water can enter room R through the leak as the air cannot escape.

Question 4

A vessel is completely filled with water. The vessel is sealed on top with a rubber membrane. In the water floats an object that is open at the bottom and contains an air bubble at the top. Pressure is to be applied to the water through the rubber membrane. What will happen?



a) The object sinks because the water pressure compresses the air bubble, reducing the buoyant force.

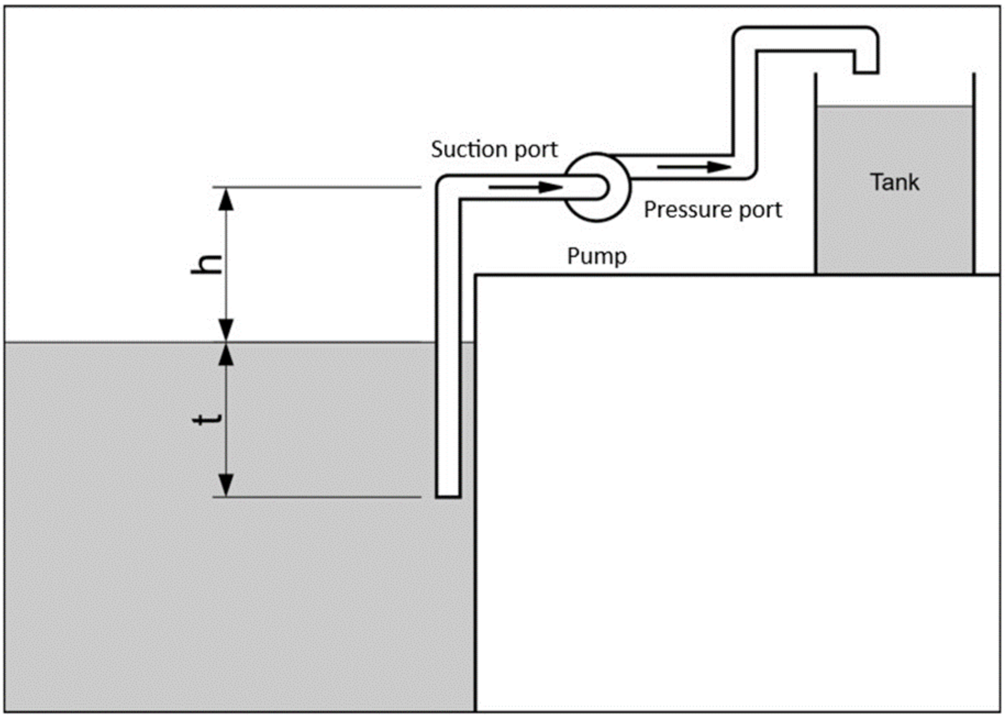
b) The object rises because the water pressure increases the density of the water, and accordingly, the buoyant force increases.

c) The object does not change its position because water cannot be compressed.

d) The object is first accelerated downwards by the pressure wave and then rises again until it is back in the same position as at the start.

Question 5

A mechanical pump with a delivery pressure of 3 bar at the pressure port is to pump water from the sea into a tank. The pump sucks water through a suction port by creating a vacuum. That means it creates a pressure lower than the ambient pressure. It delivers the water through a pressure port by creating overpressure. That means it creates a pressure higher than the ambient pressure. Ideal conditions are assumed. Friction losses, leaks, and the boiling temperature of the water need not be considered. The sketch shows a first draft of the installation. Which statement is correct?



a) The height of the suction port above sea level (h) must not be greater than the depth of the suction port below sea level (t) as the pump would otherwise not be able to suck up any water.

b) The height of the suction port above sea level (h) must not exceed 10 m as the pump would otherwise not be able to suck up any water.

c) The depth of the suction port below sea level (t) must not exceed 20 m as the pump would otherwise not be able to suck up any water.

d) The total vertical length of the suction port (h + t) must not exceed 30 m as the pump would otherwise not be able to suck up any water.

Question 6

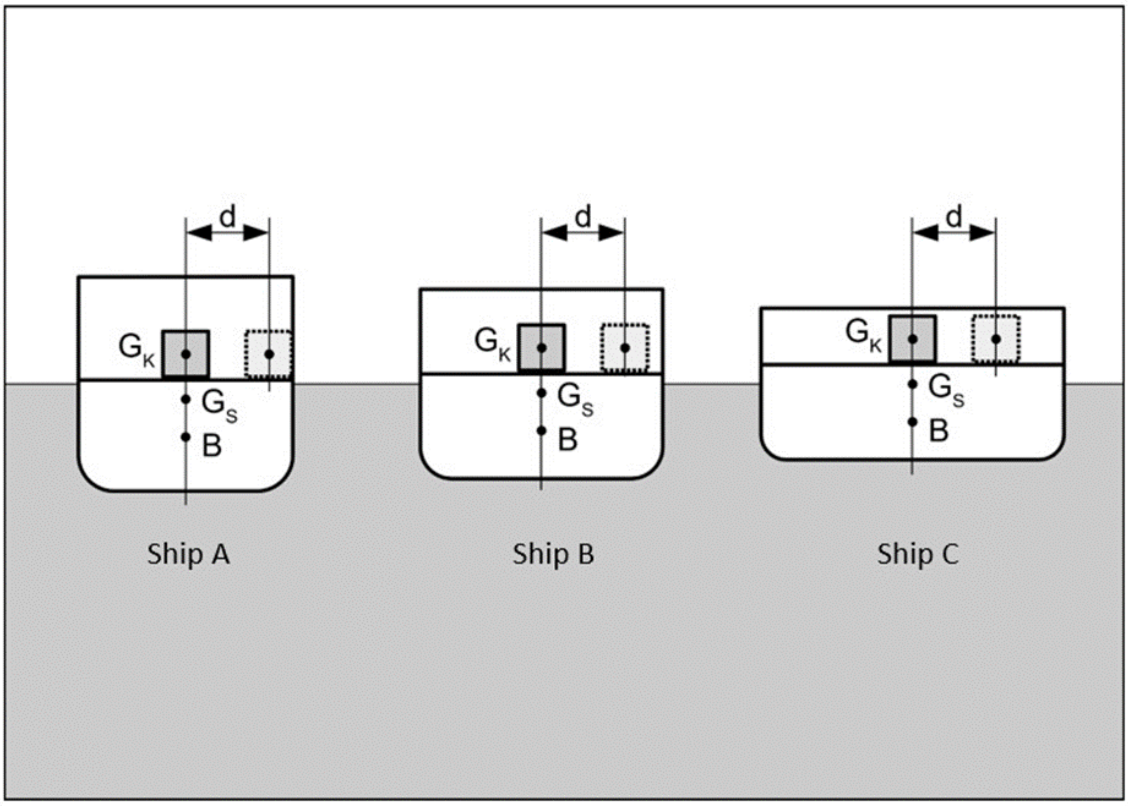
Three ships swim upright. The ships all have the same mass, so they displace the same amount of water and experience the same buoyant force. However, the ships have different cross-sections. The cross-section of the hull does not change over the length of the ship. G_S is the centre of gravity of the ship, and B is the centre of buoyancy.

All ships have a box with the same mass and dimensions loaded below deck. The centre of gravity of the box is G_K. The distances $\overline{BG_S}$ and $\overline{G_S G_K}$ are the same for all ships.

Now, on all ships, the box is moved the same amount *d* to the right.

As an initial pivot point, the centre of buoyancy *B* can be assumed.

Which statement about the effects is correct?



a) Ship A tilts the furthest.

b) Ship B tilts the furthest.

c) Ship C tilts the furthest.

d) All ships tilt by the same angle.

Subject Module	General Academic Module
Exercise 2 – Solutions	

Question 1

Solution: D

At a depth of 10,000 m, the pressure increases by about 1 bar for every 10 m. This gives approximately 1000 bar, so D is correct.

Question 2

Solution: B

A floating body displaces a mass of water equal to its own mass. With a volume of 2 m^3 in water, the displaced mass is about 2000 kg.

Question 3

Solution: B

Water can enter because the trapped air is compressed. In the simplified model, the pressure at 10 m depth is about twice atmospheric pressure, so the air volume is reduced by about half and the water rises to around 1.2 m.

Question 4

Solution: A

The object contains an air bubble that can be compressed. When pressure is applied to the water, the air bubble becomes smaller, so the object displaces less water. Therefore, the buoyant force decreases and the object sinks.

Question 5

Solution: B

A suction pump can only lift water by using the pressure difference to the atmosphere. Under ideal conditions, atmospheric pressure can lift water by about 10 m, so h must not exceed 10 m.

Question 6

Solution: A

Moving the box creates the same tilting moment in all three ships. The narrowest ship has the smallest restoring effect, so Ship A tilts the furthest.

Subject Module	General Academic Module
Exercise 3	

Optimal Order Quantity

The *optimal order quantity* is an inventory management method that aims to minimise the total costs of inventory holding and ordering. These costs typically consist of *holding costs* (the costs associated with storing inventory) and *fixed ordering costs*. Fixed ordering costs are costs that arise again with each individual order. They are constant per order because they are independent of the quantity ordered. For instance, the same amount of work is required for placing an order, whether ordering many or few products.

The key assumptions of the model are as follows:

- *Constant Demand:* The demand for products is constant and known over time. This means the demand for the product is predictable and evenly distributed throughout the year. Between two orders, inventory therefore decreases steadily: immediately after an order, it is Q; shortly before the next order, it has fallen to 0.
- *No Quantity Discounts*: The unit cost of goods remains the same, regardless of the order quantity. Bulk discounts or other price incentives are not considered in the model.
- *No Capital or Storage Limits*: It is assumed that sufficient capital and storage space are available to purchase and store large quantities.
- *Minimising Total Costs*: The objective of the model is to minimise the total costs, which consist of ordering costs and holding costs. Other business goals, such as maximising customer satisfaction or minimising lead times, are not directly considered.

The formula for calculating the optimal order quantity in a year is:

$$Q^* = \sqrt{\frac{2DS}{H}}$$

Here, Q* represents the optimal order quantity, D the annual demand in units, S the fixed ordering costs per order, and H the holding costs per unit per year. The holding costs depend on the costs for storage (e.g., rental costs) and the value of the stored goods (e.g., capital costs).

Question 1

Which statement correctly describes a central assumption of the optimal order quantity model?

a) Demand can be reliably forecast and remains stable over the year.

b) Demand fluctuates, so safety stocks are particularly important.

c) Quantity discounts are taken into account because larger orders may be cheaper.

d) The optimal order quantity is chosen so that as little storage space as possible is needed.

Question 2

What could an increase in the optimal order quantity be attributed to?

a) A reduction in holding costs per unit per year.

b) A reduction in ordering costs per order.

c) A reduction in annual demand.

d) None of the listed options would explain an increase in the optimal order quantity.

Question 3

Why does the model use the following value: $\boldsymbol{Holding\ Costs} = \frac{\boldsymbol{Q}}{\boldsymbol{2}}\boldsymbol{H}$?

a) Because $\frac{Q}{2}$ is the highest inventory level after an order.

b) Because $\frac{Q}{2}$ is the average inventory level between two orders.

c) Because only half of the products incur holding costs.

d) Because $\frac{Q}{2}$ indicates how often orders are placed per year.

Question 4

A retailer requires 1,800 units of a specific product annually. Each order placed by the retailer incurs costs of 50 euros. The cost of storing one unit of the product amounts to 2 euros per year. Given this information, which of the following options corresponds to the optimal order quantity that minimises total ordering and holding costs?

a) 150 units

b) 300 units

c) 450 units

d) 600 units

Question 5

If the ordering cost per order S is doubled and all other values stay the same, how does the optimal order quantity change?

a) It halves.

b) It doubles.

c) It quadruples.

d) It increases by a factor of $\sqrt{2}$.

Question 6

If the value per unit of the stored product increases, how might this affect the optimal order quantity?

a) It could decrease because the annual holding cost per unit might increase.

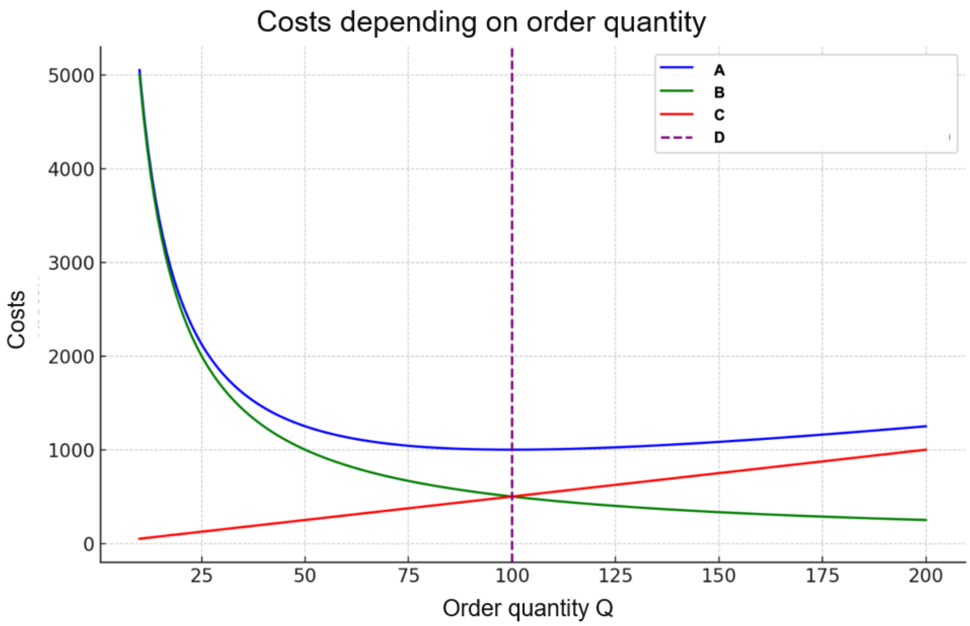
b) It could increase because the holding cost rate might increase.

c) It remains unchanged because the value per unit generally has no influence on Q∗.

d) It could increase because the fixed ordering costs per order decrease as a result.

Question 7

The following figure shows various costs (A to D) as a function of order quantity. D represents the optimal order quantity.



Which line shows the total fixed ordering costs incurred by all orders in one year?

a) Line A

b) Line B

c) Line C

d) Line D

Subject Module	General Academic Module
Exercise 3 – Solutions	

Question 1

Solution: A

This is a central assumption of the basic optimal-order-quantity-model: annual demand is known and occurs at a constant rate over time (see description of „constant demand").

Question 2

Solution: A

The optimal order quantity $\boldsymbol{Q}^* = \sqrt{\frac{\boldsymbol{2DS}}{\boldsymbol{H}}}$ increases if:

- annual demand D increases,
- ordering cost per order S increases,
- or holding cost per unit per year H decreases.

So a reduction in holding costs leads to a larger optimal order quantity, because keeping inventory becomes cheaper.

Question 3

Solution: B

In the model, as explained in the text (see: description of "constant demand"), inventory is the highest immediately after an order arrives: Q. It then decreases steadily until it reaches 0, just before the next order arrives. So the average inventory level is $\frac{Q}{2}$.

Question 4

Solution: B

Using the given formula $Q^* = \sqrt{\frac{\boldsymbol{2DS}}{\boldsymbol{H}}}$,

where D = 1800, S = 50, H = 2

So:

$$Q^* = \sqrt{\frac{2 \cdot 1800 \cdot 50}{2}}$$

$Q^* = \sqrt{90000} = 300$

Therefore, the optimal order quantitiy is 300 units.

Question 5

Solution: D

Using the given formula $\boldsymbol{Q}^* = \sqrt{\frac{\boldsymbol{2DS}}{\boldsymbol{H}}}$

Doubling S results in $Q^*_{new} = \sqrt{\frac{2D \cdot 2S}{H}}$

This means: $Q^*_{new} = \sqrt{2} \cdot Q^*$

So the optimal order quantity does not double. It increases by the square root of 2, which is about 1.41.

Question 6

Solution: A

In the optimal-order-quantity model, the optimal order quantity is calculated as:

$$\boldsymbol{Q}^* = \sqrt{\frac{\boldsymbol{2DS}}{\boldsymbol{H}}}$$

Here, H represents the annual holding cost per unit. If the value of the stored product increases, the annual holding cost per unit may also increase, for example because more capital is tied up in inventory or because insurance and risk-related costs are higher. Since H is in the denominator of the formula, a higher value of H leads to a lower value of Q∗. Therefore, the optimal order quantity could decrease.

Question 7

Solution: B

The total fixed ordering costs per year decrease as the order quantity Q increases, because larger orders mean that fewer orders have to be placed per year.

Total fixed ordering costs = annual demand Q × fixed cost per order

So, if Q increases, the number of orders per year decreases. Therefore, the total fixed ordering costs also decrease.

Subject Module	General Academic Module
Exercise 4	

Research Strategies in Social Sciences

In general, empirical social research distinguishes between *qualitative* and *quantitative* social research. Both research strategies can contribute to explaining social action. Two aspects can be differentiated here: Firstly, it can be investigated whether a certain factor relates to a certain outcome (*causal relationship*). It could be examined, for instance, whether a higher degree of education contributes to a higher income. Secondly, it could be investigated how such an outcome arises (*causal mechanism*). The focus here is on the process between cause and result. With regard to the given example, this mechanism could be that a higher education opens better professional opportunities. Quantitative and qualitative research strategies give different weights to these two aspects of explanation.

The quantitative (or theory testing or deductive) strategy aims at identifying causal relationships. This is done, for example, by conducting standardised data collection and applying statistical tests to this data. A sample of appropriate size (depending on the exact methodological parameters and statistical procedures) is central to the statistical significance.

In contrast, the qualitative (or inductive or theory generating) strategy seeks causal mechanisms. Identifying these causal mechanisms is based on the investigation of one case (single-case study) or a few cases. This also allows obtaining information about the individual conditions necessary for the mechanism to function. However, this strategy does not provide information about how widespread these conditions are.

The research process for both strategies can be divided into several phases: In the first phase, a research problem is identified, and a research question is formulated. In the second (the empirical) phase, the specification of the research design takes place. Here, for example, the period of the investigation, the investigation units, and the collection methods are determined. In the third phase, data collection and analysis of the collected data take place. In the fourth phase, the processing of the material takes place, for example, as a research report or publication. While the broad division of the phases in the research process is similar for both research strategies, the details and the ideal typical procedure differ.

Quantitative research projects have an ideal-typical procedure that runs linearly. The research design determines central decisions for the further processing of the project such as hypotheses, measurement instruments, sample and methods for data collection. Conceptual, theoretical, methodological and empirical steps are more strongly separated from one another than in qualitative projects. Subsequent modifications are not generally excluded. They must, however, be documented clearly and they have to be considered for data analyses. Some subsequent changes for instance regarding measurement instruments, samples or methods for data collection can have the consequence that data and results are only limited in their significance or comparability.

In contrast, qualitative research projects can include circular elements, meaning that phases of the investigation do not have to be clearly separated or processed in a specific order. As long as changes are clearly explained and documented, it is possible, for example, to change preconceptions or modify the methodological approach during the process.

Question 1

How can the following sentence be correctly finished? The deductive research strategy...?

a) ... requires a certain, fixed sample size, independent of research design.

b) ... focusses on the identification of causal mechanisms.

c) ... examines the relation between factors and effects.

d) ... shows how and why certain conditions influence causes and effects in single cases.

Question 2

Which of the following statements applies to the inductive research strategy?

a) It is crucial to make statistically proven statements about how well research results scale and are transferable.

b) It is crucial to examine in which way certain factors have certain consequences.

c) It is only possible to consider a single case.

d) Proof about the spread of functioning mechanisms is provided.

Question 3

Which of the following statements about conducting research projects is true?

a) In a qualitative research project, the research question can be reformulated during the process.

b) In a quantitative research project, the sample can be modified in any way after the first data analyses, if this seems to help to better examine the hypothesis.

c) In a qualitative research project, changes to the research design do <u>not</u> need to be recorded.

d) In a quantitative research project, the order of the four phases is <u>not</u> important.

Question 4

Which approach most clearly contradicts the ideal-typical process of a quantitative research project?

a) In a qualitative study on screen time and friendships, five teenagers are interviewed. During the analysis, it becomes clear that all respondents are very introverted. The research question is then refined in a clearly defined way.

b) In a qualitative study on screen time and friendships, five teenagers are interviewed first. Afterwards, further teenagers with different personality traits are deliberately included.

c) For a quantitative study with the research hypothesis “The more time teenagers spend in front of screens, the fewer friends they have”, 500 randomly selected teenagers are surveyed. During the analysis, it becomes clear that many teenagers with high screen time have many friends. The research hypothesis is then retrospectively adapted so that it fits the results.

d) For a quantitative study with the research question “What role does screen time play in teenagers’ friendships?”, two data collections are planned. After the first data collection, it is documented that girls are clearly underrepresented in the sample. Therefore, the second data collection is carried out according to a previously defined supplementary plan so that the sample becomes more balanced overall.

Question 5

A doctoral student wants to pursue two questions for her dissertation: why young people use the largest city library and what literature they are interested in. She wants to choose a mixed qualitative-quantitative approach. Which procedure might be suitable for her data collection?

a) An analysis of the media titles from 1,000 anonymous borrowing records and semi-structured interviews with 4 young people.

b) A group discussion with 3 young people and an interview with the library director.

c) A count of all books borrowed (without considering the book titles) by young people over 10 days and an interview with the largest publisher in the city.

d) A count of the young people entering the library and the evaluation of the city's population statistics.

Question 6

A master's student wants to investigate in his master’s thesis whether certain student behaviours affect productivity in seminar groups. To do this, he chooses a well-known theoretical approach from psychology. He wants to evaluate with statistical methods how far-reaching and transferable this approach is. Which statement applies to his dissertation?

a) The student has chosen a deductive approach.

b) The student must consider possible causal mechanisms in his research design.

c) The student should conduct targeted individual surveys on campus.

d) The student cannot use statistical tests because seminar groups are too different.

Subject Module	General Academic Module
Exercise 4 – Solutions	

Question 1

Solution: C

The deductive or quantitative strategy aims to identify causal relationships. It therefore examines whether certain factors are related to certain outcomes (see paragraphs 1 and 2).

Question 2

Solution: B

The inductive or qualitative strategy focuses on causal mechanisms. It examines how and why certain factors lead to certain consequences (see paragraphs 1 and 3).

Question 3

Solution: A

Qualitative research can include circular elements. This means that the research question or methodological approach may be refined during the process, as long as changes are documented (see paragraphs 4 and 6).

Question 4

Solution: C

In a quantitative research project, hypotheses should be defined before data analysis. Changing the hypothesis retrospectively to fit the results contradicts the ideal-typical quantitative process (see paragraphs 4 and 5).

Question 5

Solution: A

The borrowing records provide quantitative information about the literature young people are interested in. The interviews can provide qualitative insights into why young people use the library (see paragraphs 1 to 3).

Question 6

Solution: A

The student starts from an existing theory and wants to test how far the approach applies using statistical methods. This corresponds to a deductive research strategy (see paragraphs 1 and 2).

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